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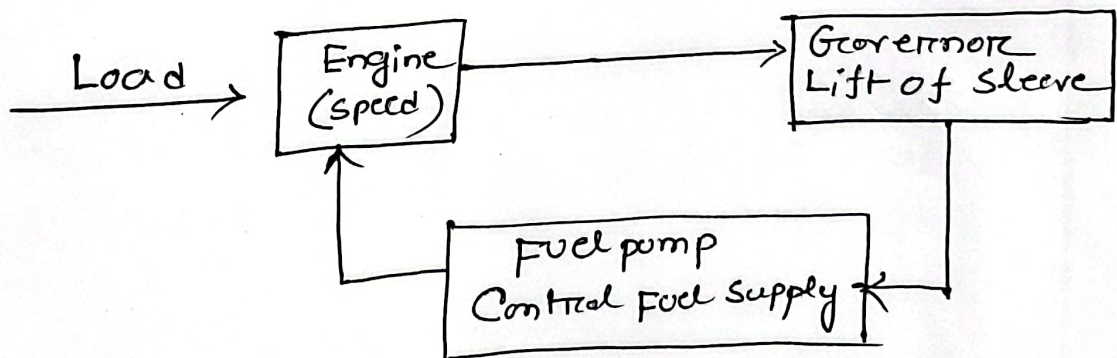
Simulation:

Simulation is the imitation of the reality.

Simulation is the representation of a real life system by another system, which depicts the important characteristics of the real system and allows experiment on it.

System:

A system is a collection of components where individual components are connected each other by interrelationships such that the system as a whole performs some specific function to solve the problem.



Activities which occur within the system are called endogenous activities.

Activities which occur in environment are called exogenous activities.

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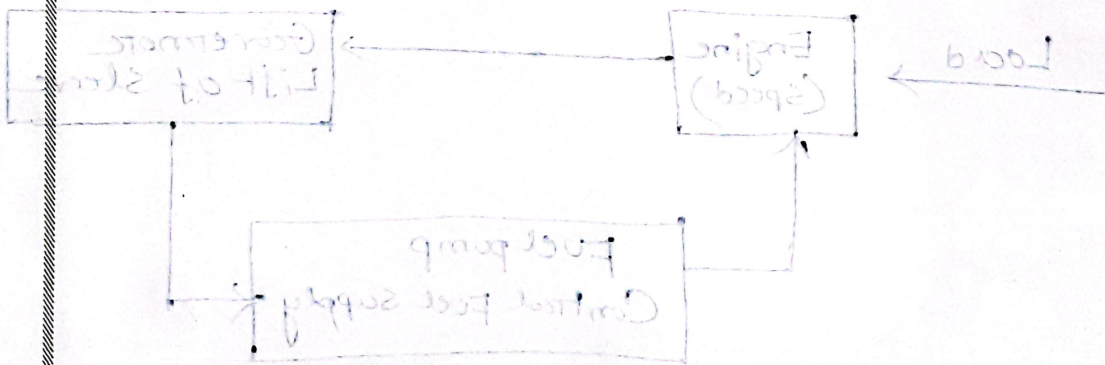
Continuous

- 1) State changes continuously with time.
- 2) Ex: The pure pursuit problem. Since target and pursuer location changes continuously.

Discrete

- 1) State changes suddenly at discrete points with time.
- 2) Ex: Inventory system, Queuing System. Demand of items of the stock occur at discrete point or customers arrive and leave the system at discrete point.

In a factory system though the start and finish of a machine are discrete points but the machine process is continuous.



Real time Simulation → is

A ~~system~~ ^{computer} that mimics real world system in real-time

Example

- 1) Aircraft cockpit simulator for the training pilots
- 2) Simulated zero gravity chambers
- 3) Windy weather app.

Simulation model → Simplified representation of real world system.

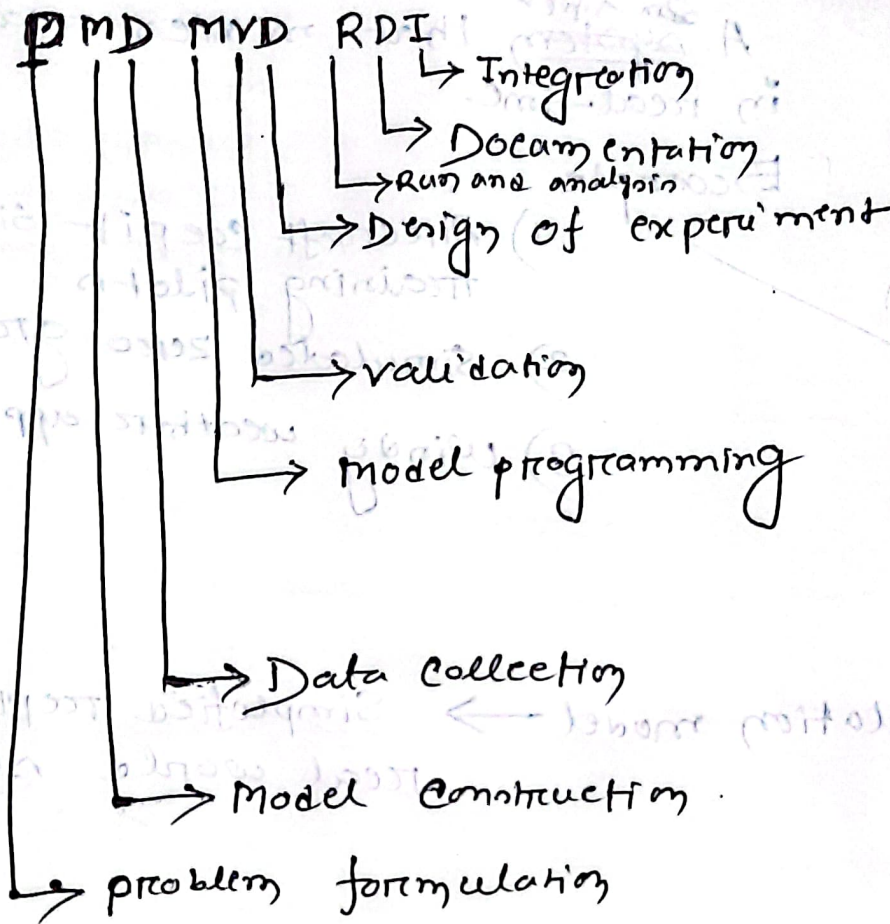
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Steps in Simulation Study →

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Problem formulation

Model construction

Data collection

Model programming

Validation

Design of experiment

Run and analysis

Documentation

Integration

Phase of a Simulation Study → PMRI

- 1) problem formulation
- 2) Model Building
- 3) Running the model
- 4) Implementation

Advantage →

- 1) વિભિન્ન પ્રણાલિના માકાર જાણે, વિભિન્ન પ્રણાલિ અમલમાં લાગુ રાખી
- 2) કોઈ વિભિન્ન પ્રણાલિ નિર્માણ કરવામાં આવે તો તે સરળતાથી અમલમાં લાગી શકે છે।
ઉદાહરણ: હાલમાં માટેના મુદ્દાઓ બાદમાં રાખવામાં આવે છે।
- 3) પ્રણાલિના અમલમાં જે કોઈ ભૂલો થાય છે તે સરળતાથી ઓળખી શકાય છે।
ઉદાહરણ: કોઈ ભૂલ થાય તો તે સરળતાથી ઓળખી શકાય છે।
- 4) પ્રણાલિના અમલમાં જે કોઈ ભૂલો થાય છે તે સરળતાથી ઓળખી શકાય છે।
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- 5) પ્રણાલિના અમલમાં જે કોઈ ભૂલો થાય છે તે સરળતાથી ઓળખી શકાય છે।
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ઉદાહરણ: Aircraft Carrier
Military Squad

1) આશ્રિત સ્થાન સ્થાન

2) ഗ്രാമപഞ്ചായത്ത് നിയമസഭയിൽ ചേർന്ന് തീരുമാനം എടുക്കുക

4) අනුමැතිය ලබාදීම සඳහා
 5) අනුමැතිය ලබාදීම සඳහා
 6) අනුමැතිය ලබාදීම සඳහා

8) $\frac{1}{x^2} = x^{-2}$ $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

[illegible]

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1) Manufacturing,

2) Business

3) military

4) Health care applications

5) Communication Application

6) Computer application

7) Economic Application

8) Transportation Application

9) Biological Application

Computer Science

→ network design

→ network reliability evaluation

4. sizing system buffer

↳ Originating hardware

- Software simulation.
- test under different condition

Game development

→ Robotics

↳ Algorithm analysis.

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Analytical Solution



Simulation

- 1) Deriving exact mathematical formula
 - 2) ~~Need~~ used Algebraic manipulation method
 - 3) Result are closed form equation
 - 4) Give exact accuracy
 - 5) Suitable for simple system
 - 6) Example:
Find area of a circle
- 2) Numerically approximate the solution
 - 2) Computer based modeling method
 - 3) Result are Numerical values or graph
 - 4) Depends on simulation parameters and steps.
 - 5) Suitable for complex system
 - 6) Traffic flow analysis through a city

Disadvantages of Analogue Simulation

- 1) Accuracy
- 2) Security
- 3) Scalability
- 4) Time Consuming
- 5) Flexibility
- 6) Cost
- 7) Data Collection

~~7/6/2020~~

Effective Random number generator → 7

- 1) Independent
- 2) Unpredictable
- 3) Efficient
- 4) Statistical Sound
- 5) portable
- 6) Fast and cost effective
- 7) Sufficiently Long cycle

767
2125

System Study ways →

- ① Experiment with actual system
- ② Experiment with a model of the system
 - i) physical model
 - ii) mathematical model
 - A. Analytical Solution
 - B. Simulation

Monte Carlo method

new school
trial
m/m

(It is a technique of experimental sampling with random numbers) or the method of trials which can be used to solve many problems, which are otherwise difficult or impossible to solve.

Normally Distributed Random numbers →

A set of random values that follow a normal (Gaussian) distribution meaning they have a bell-shaped curve.

properties:

★ Symmetry

★ Bell shaped curve

★ Mean = median = mode

★ Unbounded

Application →

★ Statistical analysis

★ Machine learning

★ Financial modeling

★ Quality Control

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Define Autocorrelation →

The uniformity test of random numbers is only a necessary test for randomness, not a sufficient one. A sequence of numbers may be perfectly uniform and still not random. For example the sequence .1, .2, .3, .4, .5, .6, .7, .8, .9, .1, .2, .3, ... would give a perfectly uniform distribution with the Chi-square value as zero. But the sequence can by no means be regarded as random.

The numbers are not independent as the occurrence of one number say .3 decides the next, which is to be .4 etc. This defect is called serial autocorrelation of adjacent pairs of numbers.

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Random Numbers →

A number that is drawn from a uniformly distributed random variable between some specified intervals and they have equal probability of occurrence.

pseudo Random Numbers → A random numbers

that is generated by using some arithmetic operations.

Disadvantages:

- * may not be uniformly distributed
- * may not be continuous.
- * mean may be too high or too low.
- * variance may be too high or too low.
- * They are may be cyclic pattern.

Method:

* Congruence or Residue

3 type → additive $\Rightarrow r_{i+1} = (r_i + b) \text{ modulo } m$
multiplicative $\Rightarrow r_{i+1} = ar_i \text{ modulo } m$
mixed $\Rightarrow r_{i+1} = (ar_i + b) \text{ modulo } m.$

* Arithmetic Congruential $\Rightarrow r_{i+1} = (r_{i-1} + r_i) \text{ modulo } m$

Disadvantages of Congruence method →

- 1) Not possible to generate a non repeating sequence
- 2) Numbers obtained are not truly random. Because the number can be predicted rather computed from ri and whole string is reproducible.

Why Testing ~~and~~ Uniformity and Independence are necessary →

- 1) Ensure values are evenly distributed
- 2) prevent biases that could affect security
- 3) value is not dependent on previous value
- 4) prevents patterns that could lead to predictability

ensure evenly distributed
prevent biases

prevent patterns

problem Statement →

Consider a single server queuing system for which the interarrival times $A_1, A_2, \dots$ are independent and identically distributed (IID) random variables.



A departing Customer

IID

independent and identically distributed



Server



A Customer in service



Customers in queue



An arriving Customer

A Customer who arrives and finds the server idle enters service immediately, and the service times $S_1, S_2, \dots$ of the successive customers are IID random variables that are independent of the interarrival times.

A customer who arrives and find the server busy joins the end of a single queue.

Upon completing service for a customer, the server chose a customer from the queue (if any) in a FIFO manner.

performance metrics $\rightarrow$

1) Expected average delay: For a single run simulation, resulting in customer delays $D_1, D_2, \dots, D_n$ an obvious estimator of $\bar{d}(n)$ is

$$\bar{d}(n) = \frac{\sum_{i=1}^n D_i}{n}$$

2) Expected average number of customer in the queue $\rightarrow$

$P_i \rightarrow$ This is the portion of time during the simulation when there were exactly i customers in the queue

Ex: If simulation time 100 min, and for 20 min there were 2 people in the queue then $P_2 = \frac{20}{100} = 0.2$

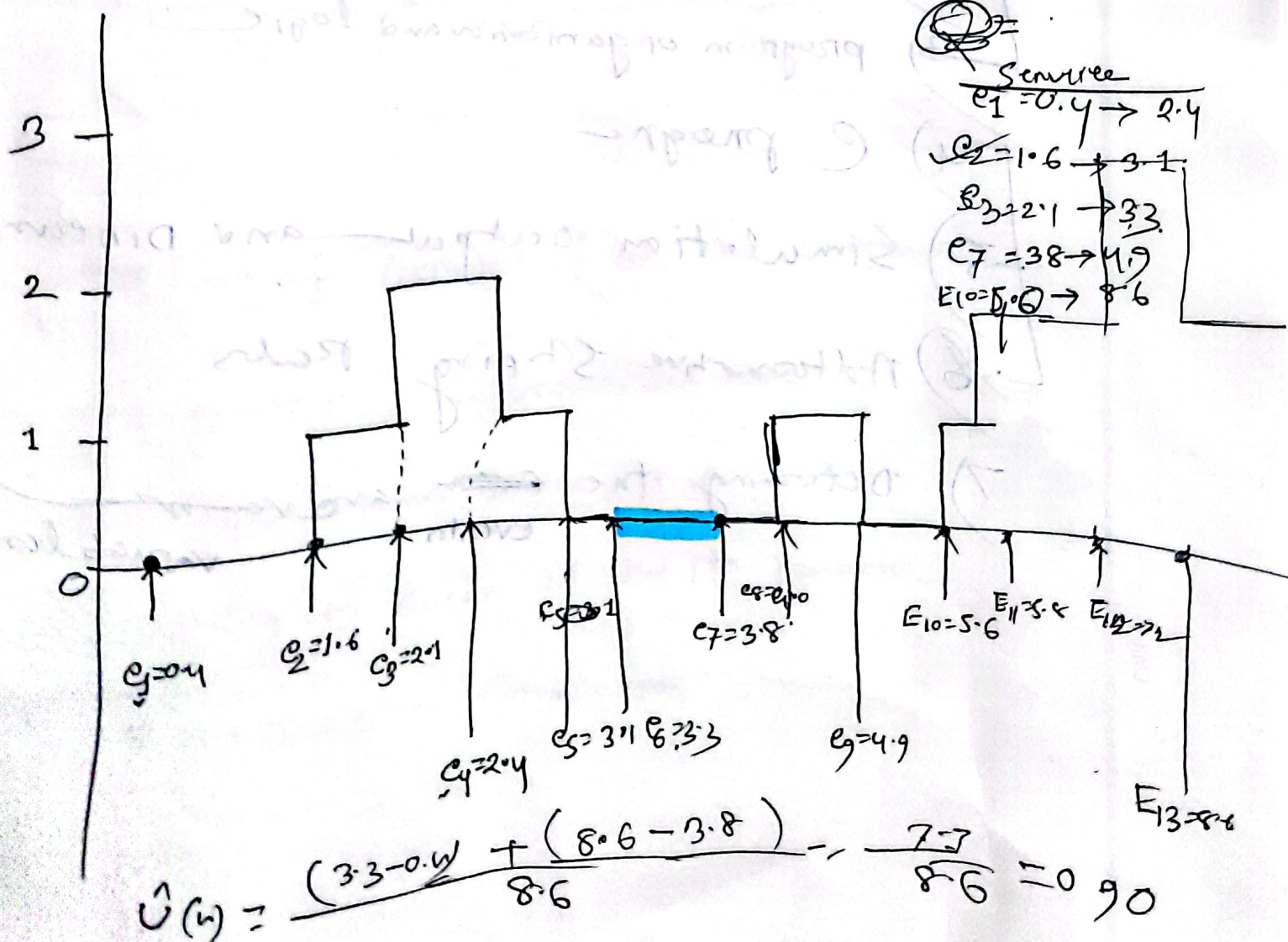
T_i : This is the total time the Queue has exactly i customers (20)

$T(n)$: Total simulation time (10000)

$$P_i = \frac{T_i}{T(n)}$$

Average number of Customers in the queue

$$\hat{q}(n) = \sum_{i=0}^{\infty} i \cdot P_i = \frac{\sum_{i=0}^{\infty} i T_i}{T(n)}$$



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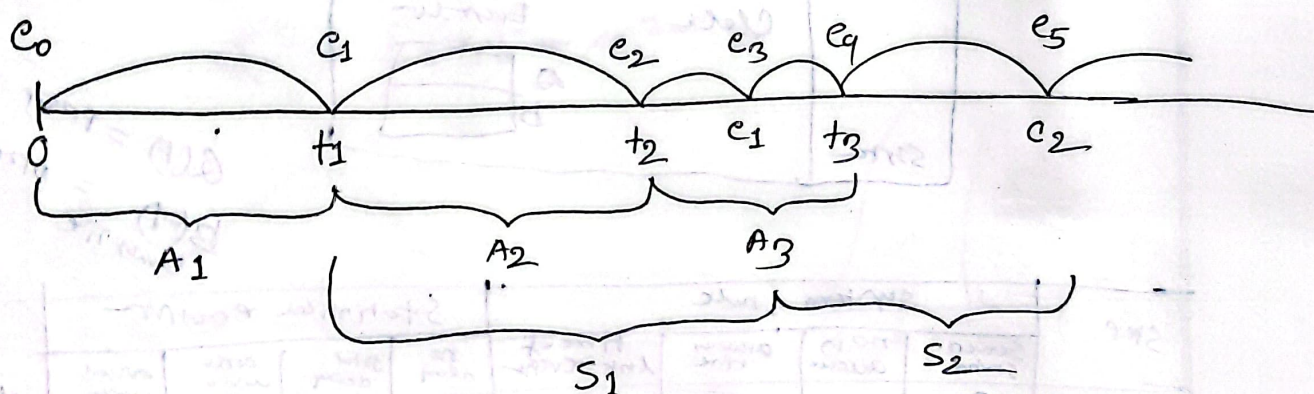
This indicating that the server was busy about 90 percent of time during this simulation

Steps of simulation for a single server queuing system → ⑦

- 1) problem statement
- 2) Intuitive explanation
- 3) program organization and logic
- 4) @ progne
- 5) Simulation output and discussion
- 6) Alternative stopping rules
- 7) Determining the ~~even~~ events and ~~variables~~ variables

Explain the next event time advance approach illustrated for the single server queuing system.

Time at which customer departs = arrival time + service time + delay time after getting service



t_i = Time of Arrival of the i th customer

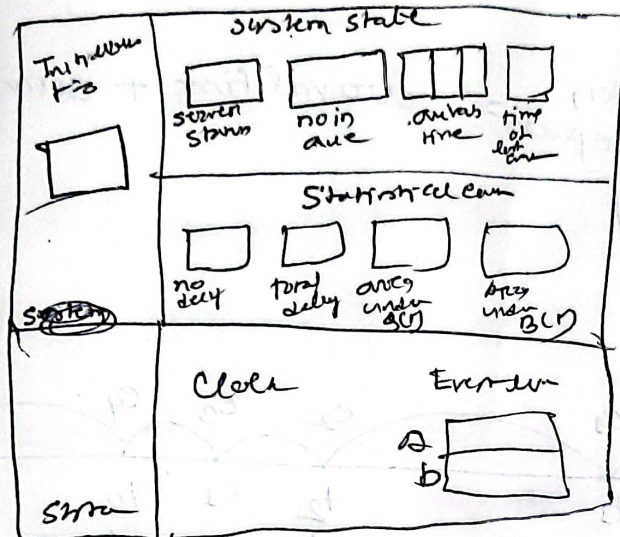
$A_i = t_i - t_{i-1}$ Intercarrival time

S_i = Service time of the i th customer

D_i = Delay in Queue of the i th customer

$C_i = t_i + D_i + S_i$ = Completion time

Intuitive Explanation



$Q(t) = \text{prev no in queue}$
 $B(t) = \text{prev server status}$
 Event time

Step	System state				Statistical count				Event	
	Server status	no in queue	arrival time	Time of left event	no delay	total delay	area under $a(t)$	area under $B(t)$	Clock	Event
Initiation 	0	0		0	0	0	0	0	0	A 0.4
Arrival time 0.4	1	0		0.4	1	0	prev no in queue (0)	0	0.4	1.6
Arrival 1.6 0.6	1	1	1.6	1.6	1	0	$0 + 0.6 = 0.6$	$0 + 1 = 1$	1.6	2.1
Arrival 1.6 1.6	1	2	1.6	2.1	1	0	0.5	1.7	2.1	3.8
Arrival 1.6 2.4	1	1	2.1	2.4	2	1.6	$0.5 + 1.6 = 2.1$	2.4	2.4	3.1
Arrival 1.6 3.1	1	1	2.1	2.4	2	1.6	0.3	2.1	2.4	3.8
Arrival 1.6 3.8	1	1	2.1	2.4	2	1.6	0.3	2.1	2.4	3.8

Variance:

The variance is a measure of the dispersion of a random variable about its mean, as seen below figure the larger the variance, the more likely the random variable is to take on values far from its mean.

$$\sigma^2 = E(x_i)^2 - \mu_i^2$$

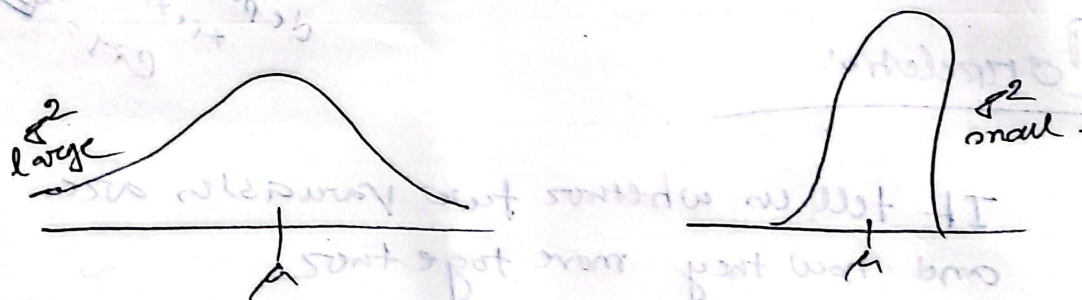


Fig: Density functions for continuous random variables with large and small variance.

Covariance:

The Covariance between the random variables X_i and X_j which is measure of their (linear) dependence, will be denoted by $C_{ij} = E(X_i X_j) - \mu_i \mu_j$

measure of the
dependence of X_1 with
 X_2
 $C_{ij} = E(X_i X_j) - \mu_i \mu_j$

Correlation:

It tell us whether two variables are related and how they move together

Why Stochastic process

A stochastic process is used to simulate situations where outcomes are uncertain or random. It helps predict possible results when we can't know for sure what will happen, but we can estimate based on probabilities

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Example

Imagine you are trying to guess how many customers visit a shop each day. Some

On days there are 20, other days 50, and it changes randomly. A stochastic process helps

simulate these changes, giving you a range of possible outcomes instead of just one fixed number.

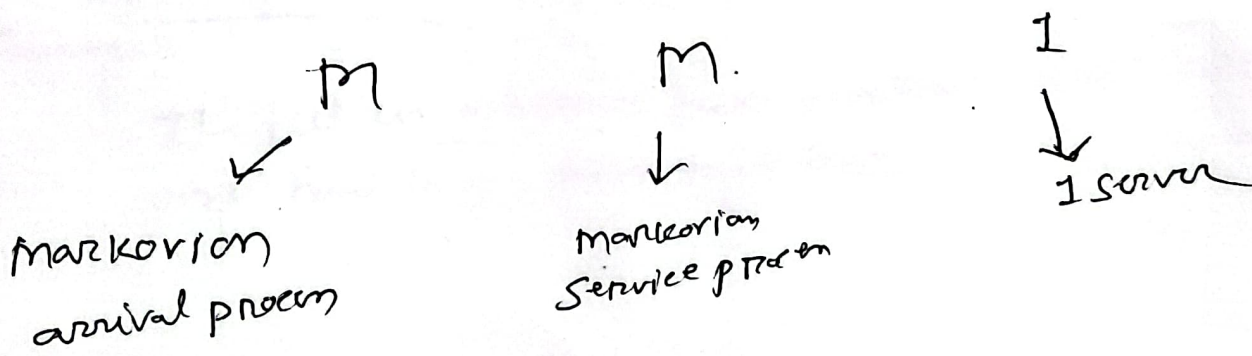
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M/M/1 model:

The M/M/1 model is a queueing process in which customers arrive at one server and wait in a queue (if necessary) until the server is available. Customers are serviced in the order in which they arrive. The server services at most one customer at a time. There is no limit to the number of customers who can wait in the queue.



In the M/M/1 queue the delays $D_1, D_2, \dots$ are positively correlated because the service times of earlier customers affect the wait times of later customers. When

one customer experiences a long delay, the next customer is likely to experience a longer delay as well.