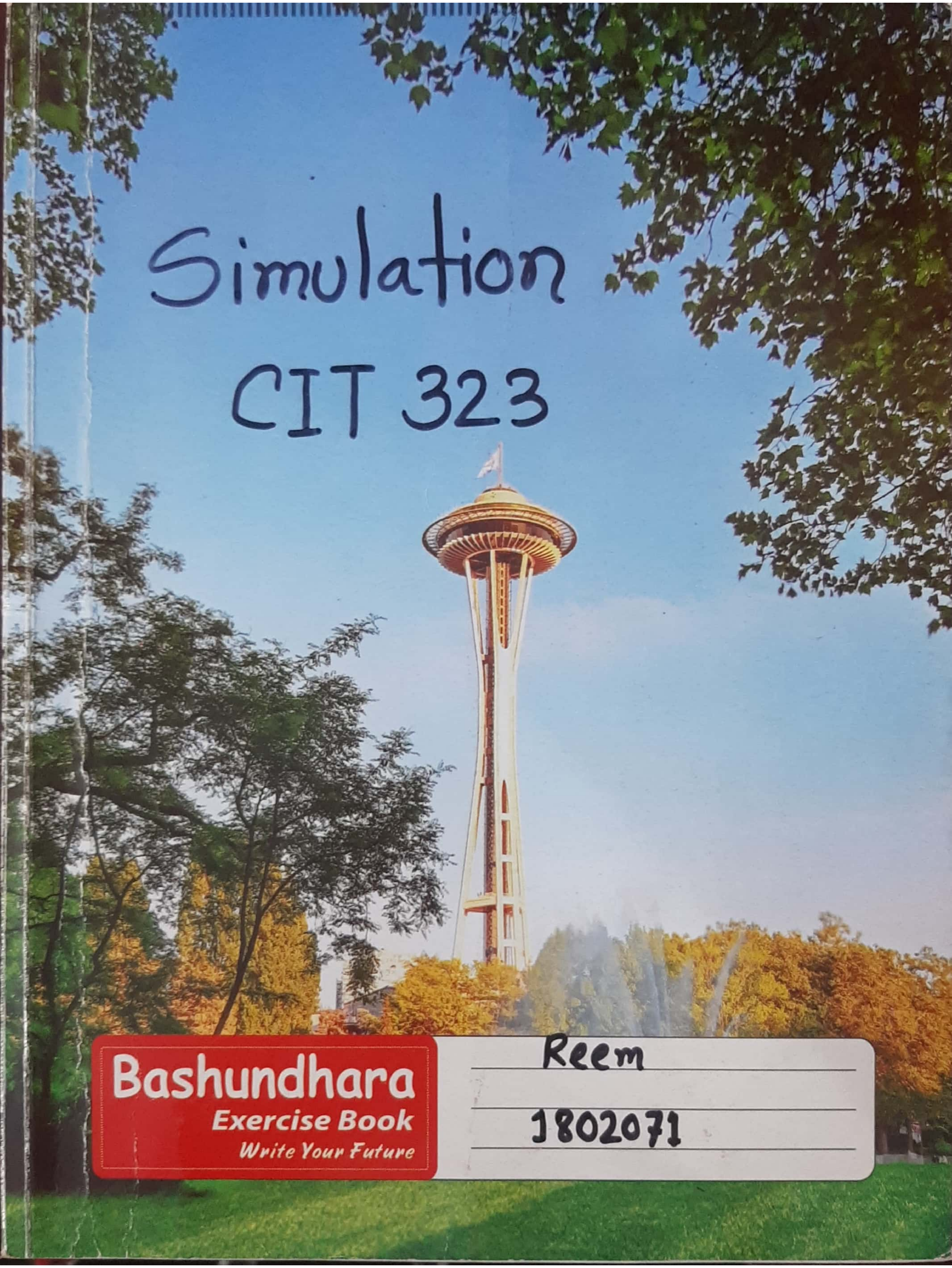
Simulation

CIT 323



Bashundhara
Exercise Book
Write Your Future

Reem

1802071

Reference Book

i) System simulation - 2nd Edition

D. S Hirca

Lecture-01 (MMR)

Date - 27/02/2023

CIT 323

Simulation and Modeling

Classroom code - 7sm3lx3
Simulation

Classroom code - 5oxofom
Operating System

Simulation - A simulation is the ~~relation~~ imitation of the operation of a real world process or system over times.

Computer simulation - Computer simulation is the process of mathematical modelling, performed on a computer, which is designed to predict the behavior or outcome of a real-world or physical system.

Chapter 01

System - Collection of interrelated components wherein individual components are constrained by connecting inter-relationships such that the system as a whole fulfills some specific functions in response to varying demands.

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System: An aggregation or assemblage of objects joined in some regular interaction or interdependency.

System 2 types:

1) Endogenous activity (system এর ভিতরে হয়) closed system
2) Exogenous activity (system এর বাইরে হয়) open system

Entity: Component of interest and the property of interest is called attribute.

Queue: situation where entities wait for something to happen

Creating: causing an arrival of new entity.

Scheduling: process of assigning events to the existing entities.

Random variable: whose quantitative value changes at random

Random variate: generated by using random numbers along with its probability density function.

Distribution - random variable এর probabilistic feature define করে

Table 1.1

System Classification: 2

Continuous system - sinusoid curve follow করে; state of the system changes continuously with time

Discrete system - state of the system changes abruptly at discrete points in time.

Simulation Models:

I) Physical Model (Physical objects are substituted for real things)
- Static (doesn't change with time)
- dynamic (changes over time)

II) Mathematical Model (Comprise of symbolic notations and mathematical equations to represent the system)
- Static → Numerical (requires applications of numerical methods) / Analytical (solved by employing analytical techniques)
- dynamic → Numerical → System simulation / Analytical

Deterministic models: have a known set of inputs, which result into a unique set of outputs. occurance of events in certain and not dependent on chance.

Stochastic models: one or more random variables, which lead to random outputs. The outputs in such a case are only estimates of true characteristics of the system.

System simulation: The technique of solving problems by observations of the performance, over time, of a dynamic model of the system.

Computer simulation: process of mathematical modelling, performed on a computer, which is designed to predict the behavior of or the outcome of a real-world or physical system.

Physical simulation: refers to the simulations in which physical objects are substituted for real things.

Interactive simulation: special kind of physical simulation in which the human operator interacts with the physical model.

Real time simulation: The computer model runs at the same rate as the actual physical system.

* Simulation VS Analytical Methods

i) Simulation gives specific solutions depending upon the input conditions and characteristics of the system

Analytical methods give general solutions

ii) Results obtained by simulations are approximate while results obtained by analytical methods are exact.

iii) Simulation can handle all sorts of problems, analytical methods can so handle a limited range of problems.

* When to use simulation?

1. For experiments with the internal interactions of a complex system

2. To experiment with new designs and policies, before implementing them

3. To ~~very~~ verify the results obtained by analytical methods and to reinforce the analytical techniques.

4. In determining ~~of~~ the influence of the changes in input variables on the output of the system.

5. In suggesting modifications in the system, under investigation for its optimal performance.

* Steps in simulation study

1. Problem formulation - must ensure problem being described is clearly understood

2. Model construction - no standard rules but some guidelines for building successful and appropriate models.

3. Data collection - The kind of data to be collected depends upon the objectives of the study.

4. Model programming - The translation of the model into a computer recognizable format i.e general or special simulation language BASIC, FORTRAN, C, C++

5. Validation - It is essential to ensure that the model is an accurate representation of the system that have been modeled.

6. Design of Experiment - To make the results meaningful, it is essential that simulation experiment be designed in such a way that the results obtained are within some specified tolerance limit and at a reasonable level of confidence.

7. Simulation run and analysis - The simulation program is run as per the design; The results are obtained and analyzed to estimate the measurement of the performance of the system. Based on the results a decision is made whether or not any modification is needed.

8. Documentation - Documentation is necessary as the program can be used by the same or different analyst in future.

9. Implementation - There won't be any problems in the implementation of the simulation program, if the user is fully conservant with the model, and understands the nature of its inputs and outputs and underlying assumptions.

* Phases of Simulation study

Phase-1: Problem formulation

Phase-2: Model building (model construction, data collection, programming, validation)

Phase-3: Running the model (experimental design, simulation run and analysis)

Phase-4: Implementation (documentation, implementation)

* Advantages of Simulation

1. helps to learn about a system

2. many managerial decision making problem solve

3. experimenting

4. compress the performance of a system over years into a few minutes of computer running time.

5. management can foresee the difficulties and bottlenecks

6. can be easily understood by operating personnel and non technical managers.

7. simulation models are comparatively flexible and can be modified

8. extensive computer packages are available, making it very convenient

9. simulation technique is easier and can be used for wide range of situations

10. good tool for training for ~~complex~~ operation of complex system.

* Limitations of Simulation Technique

1. doesn't produce optimum results
2. quantification of variables is difficult
3. difficult for very large and complex problems
4. costly method
5. too much rely on simulation, simple situations can be solved by analytical methods.

* Areas of Application

i) Manufacturing

ii) Business

iii) Military

iv) Healthcare applications

v) Communication applications

vi) Computer applications

vii) Economic applications

viii) Transportation applications

ix) Environmental applications

x) Biological applications

Lecture-03 Date- 05/03/2023

Chapter 02

Monte Carlo Method

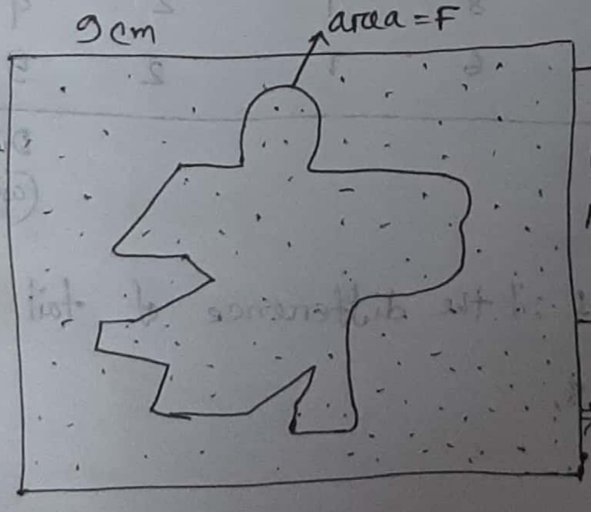
First use- nuclear explosion during world war II

Invented by - John von Neuman

Named after - casino town called Monaco

Monte Carlo method - A method which uses randomness to solve problems that might be deterministic in principle.

Ex-2.1: * Area of Irregular Figure



→ area = A

$M = 20$

$N = 50$

$$\frac{M}{N} = \frac{F}{A}$$

$$\Rightarrow \frac{20}{50} = \frac{F}{81}$$

$$\Rightarrow F = 32\ cm^2$$

যেকোন known shape এ enclose করতে হবে

Uniformly distributed r. variable

Independent r.v

যত বেশি random variable generate হবে তত বেশি accurate হবে

Figures ~~volume~~ area = F

Assumed known shape ~~volume~~ Area = A

$P = (x, y)$

x = rand(); y = rand();

M = points inside irregular figure

N = points inside square

$$P_1(x) = 0.96 \times 9 = 8.64$$

$$P_1(y) = 0.42 \times 9 = 3.78$$

0.00
. . . .
0.99

here 0.96 and 0.42 are random numbers

Ex-2.2: * Gambling Game

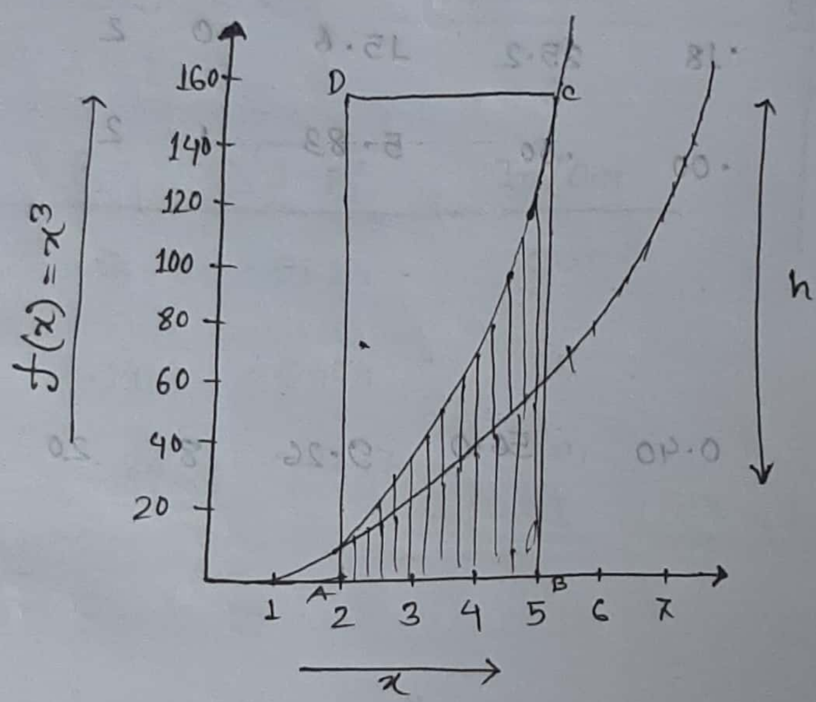
Game No.	SL No.	RN	H/T	Cumulative H	Cumulative T	Diff
1	1	5	T	0	1	1
	2	9	T	0	2	2
	3	3	H	1	2	1
	4	6	T	1	3	2
	5	4	H	2	3	1
	6	8	T	2	4	2
	7	6	T	2	5	3

Diff = 3 → win (within 8 toss)

For each flip \$1 ; if the difference of tail tossed and head tossed is 3, gets \$8,

Ex-2.3 : Numerical Integration by Monte Carlo Method

$$y = x^3 \; ; \quad I = \int_2^5 x^3 dx = \left[\frac{x^4}{4}\right]_2^5 = 152.25$$



যদি $y \leq x^3$ হয়, তাহলে M এর মান বাড়তে থাকবে

$$I = \frac{M}{N} \times A$$

$$= \frac{8}{20} \times (\underline{140 \times 3})$$

$$= 168.00$$

M = no. of points inside irregular shape

N = random no. of points inside square

I = shaded area

* Simulation of $I = \int_2^5 x^3 \, dx$

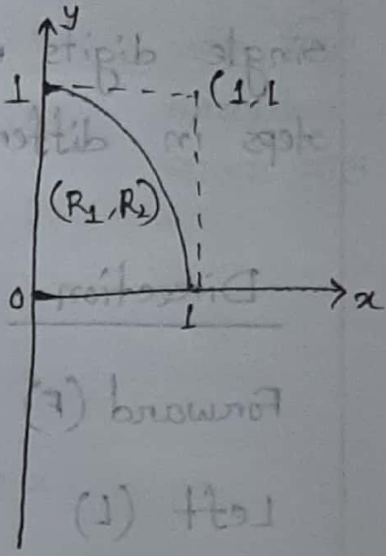
RN	x .1 (R)	RN	y 140 (R)	x^3	M	N
22	2.2	.57	79.8	10.65	0	1
25	2.5	.18	25.2	15.6	0	2
18	1.8	.00	.00	5.83	1	2
-	-	.	.	-	-	-
-	.	.	.	.	.	.
21	2.1	0.40	56.0	9.26	8	20

$$\frac{8}{20} \times 140 \times 3 = 168.00$$

$R = 152.25$

Ex-2.4* Finding the value of π

$$\frac{\pi r^2}{4} = \frac{\pi}{4} \text{ (unit circle)}$$



$$x^2 + y^2 \leq 1$$

R_1	R_2	$\sqrt{1-R_1^2}$	In/Out
.82	.95	.5724	out
.34	.14	.9404	in
.20	.84	.9747	in

$$R_1^2 + R_2^2 \leq 1$$

$$R_2 \leq \sqrt{1-R_1^2}$$

$$\frac{\pi}{4} = \frac{M}{N}$$

$$\pi = \frac{M}{N} \times 4$$

$$= \frac{31}{40} \times 4$$

$$= 3.10$$

Lecture-04 Date - 09/03/2023

Ex- 2.5 : Random Walk Problem

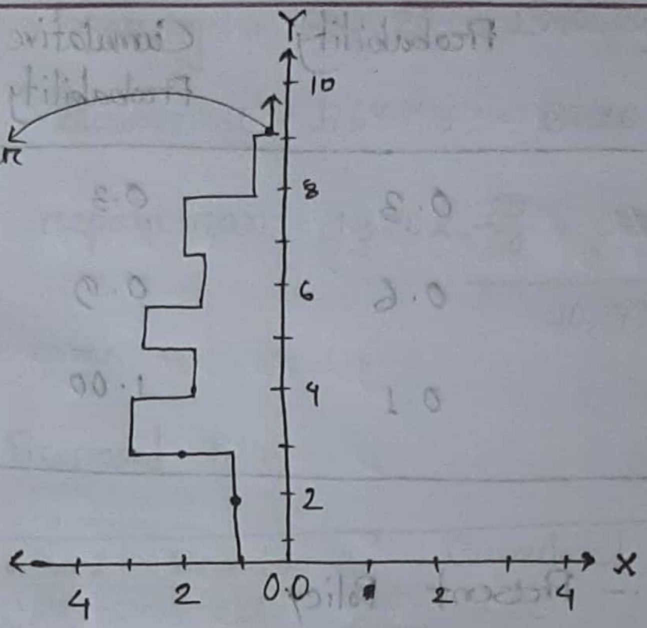
single digits random numbers can be allocated to steps in different directions

Direction	Probability	Random Numbers
Forward (F)	0.5	0, 1, 2, 3, 4
Left (L)	0.3	5, 6, 7
Right (R)	0.2	8, 9

* Simulation

Step	Random No.	Direction	Position x	Position y
1	6	L	-1	0
2	2	F	-1	1
3	0	F	-1	2
4	6	L	-2	2
5	8	R	-1	2
6	5	L	-2	2
7	7	L	-3	2

CamScanner



may be after 50 steps

Probability switch করে দিলে একটা circular area পাওয়া যাবে

Ex-2.6 : Reliability Problem

Bearing Life (Hours)	Probability	Cumulative Probability	Random Digits Assignment
1000	0.10	0.10	01 - 10
1100	0.14	0.24	11 - 24
1200	0.24	0.48	25 - 48
1300	0.14	0.62	49 - 62
1400	0.12	0.74	63 - 74
1500	0.10	0.84	75 - 84
1600	0.06	0.90	85 - 90
1700	0.05	0.95	91 - 95
1800	0.03	0.98	96 - 98
1900	0.02	1.00	99 - 00

Delay Time (Minutes	Probability	Cumulative Probability	Random Digits Assignments
4	0.3	0.3	1-3
6	0.6	0.9	4-9
8	0.1	1.00	0

* Simulation -- Present Policy

Bearing 1					Bearing 2					Bearing 3				
RD	Life	CLK	RD	Delay	RD	Life	CLK	RD	Delay	RD	Life	CLK	RD	Delay
25	1200	1200	9	6	65	1400	1400	5	6	94	1700	1700	7	6
82	1500	2700	8	6	91	1700	3100	0	8	87	1600	3300	0	8
		≥30,000												
Total Delay = 126 min					136 min					122 min				

bearing replaced = 23 x 3 = 69

time to change bearing = 69x 20 = 1380 min

total delay = 384 min

total down time = 384 + 1380 = 1764 min

Cost of bearings = 69 × 20 = 1380.00

Cost of downtime = 1764 × 5 = 8820.00

Cost of repairman = $1380 \times \frac{25}{60}$ = 575.00

10,775.00

* Simulation - Proposed Policy

RD1	Bearing 1 Life (Hrs.)	RD2	Bearing 2 Life (Hrs.)	RD3	Bearing 3 Life (Hrs.)	First Failure (Hrs.)	Cumulated Life (Hrs.)	RD	Delay (minutes)
26	1200	65	1400	92	1700	1200	1200	4	6
82	1500	91	1700	87	1600	1500	2700	6	6
	1100		1200		1400	1100	3800	7	6
	1100		1400		1400	1100	4900	3	4

Total delay = 148

Replacement = 25 × 3 = 75 bearings

Changing time = 25 × 40 = 1000 ; Total downtime = 1000 + 148 = 1148 mi

Cost of Bearings = 75 × 20 = 1500.00

Cost of downtime = 1148 × 5 = 5740.00

Cost of repairman = $1000 \times \frac{25}{60}$ = 416.67

7656.67

Proposed policy can save 3118.33 over a period of 30,000 hours.

Lecture- 05 Date-16/03/2023

Random numbers → i) true r.n ii) pseudo r.n

Normally distributed random numbers

$$y = \mu + \sigma\left(\sum_{i=1}^{12} r_i - 6\right), \quad i = 1, 2, 3, \ldots, 12$$

y = random variate

μ = mean

r =

σ = standard deviation

Alternative method where random no. are used –

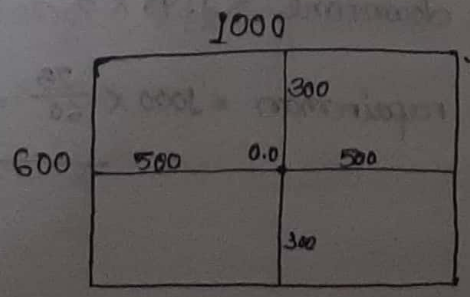
$$y = \mu + \sigma z$$

z = random no.

Normal random no. → distribution- gausian distribution

curve – bell shaped

Ex-2.7: * Application of Random Normal no.



A normal random variable X is, $\quad$ μ = mean

σ = standard deviation

Z = random normal no.

$$X = \mu + \sigma Z$$

If x, y are the co-ordinates of a point, then,

$$x = \mu_x + \sigma_x z_x$$

$$y = \mu_y + \sigma_y z_y$$

we know,

$\sigma_x = 500$

$\sigma_y = 300$

z_x, z_y = normal random no,

taking the center point's co-ordinates, the area as 0,0 then $\mu_x = \mu_y = 0$

$$x = \sigma_x z_x$$

$$y = \sigma_y z_y$$

$$\therefore x = 500 z_x$$

$$y = 300 z_y$$

* Simulation of Bombing Problem

Bomb Strike	RNN	x	RNN	y	Result
1	0.23	115	0.24	72	Hit
2	-1.16	-580	-0.02	-6	Miss
3	0.39	195	0.64	192	Hit
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

No. of Hit = 10

No. of Miss = 10

% No. of strikes on target = 50%

CamScanner

Monte carlo method = deterministic

Random normal no. = stochastic

* Difference – Monte carlo vs stochastic simulation

Lecture-06 Date- 25/03/2023

Chapter 03

Simulation of Continuous Systems

Discrete system- abruptly changes

Continous system - changes continously, generally with time
- time is not essential parameter

Examples: i) recursive procedures of solving problems

ii) determining value of π by simulation

iii) solving an integral by simulation

Continuous System -

i) progress of chemical reaction

ii) flight of a missile

iii) a servo system

→ dynamic systems

Simulation of continuous system - is the process of carrying out the computations with a detailed insight into the system.

3.2 Pure Persuit Problem

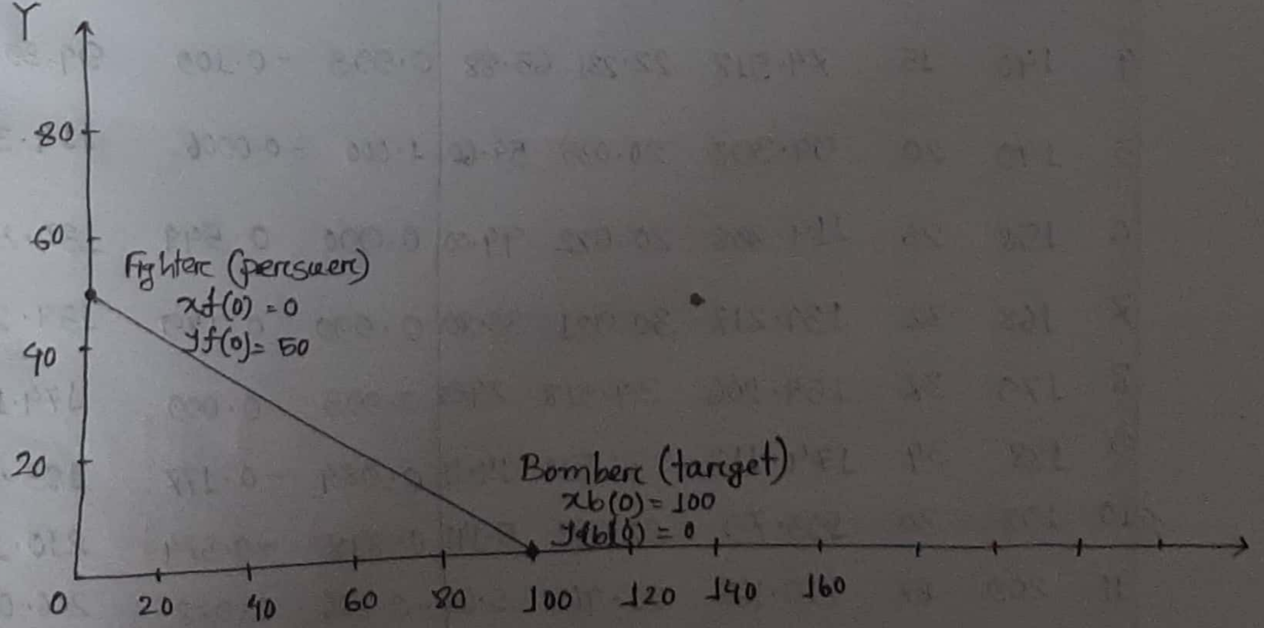
The target and persuer fly in the same horizontal plane, and stay in that plane during the period of persuit. The fighter's speed S is constant at 20 km/min, while the target's path is specified as a function of time as below:

Time (t) :	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$x_b(t)$:	100	100	120	129	140	149	158	168	179	188	198	209	219	226	234	24
$y_b(t)$:	0	3	6	10	15	20	26	32	37	34	30	27	23	19	16	14

fighter aircraft position x_f, y_f (0, 50) at time, t = 0

interval = 1 min ; shoots when distance = 10 km

persuit is abandoned if not shoot within 12 minutes of persui



$$\text{Dist}(t) = \sqrt{(yb(t) - yf(t))^2 + (xb(t) - xf(t))^2}$$

if θ = the angle with x-direction,

$$\sin\theta = \frac{yb(t) - yf(t)}{\text{Dist}(t)}$$

$$\cos\theta = \frac{xb(t) - xf(t)}{\text{Dist}(t)}$$

Co-ordinates at time $(t+1)$,

$$xf(t+1) = xf(t) + S\cos\theta$$

$$yf(t+1) = yf(t) + S\sin\theta$$

t	xb	yb	xf	yf	Dist(t)	Cosθ	Sinθ	x(t+1)	y(t+1)
0	100	0	0	50	111.8033	0.894	−0.447	17.88	41.06
1	100	3	17.8	41.06	90.5836	0.907	−0.420	35.94	32.66
2	120	6	35.94	32.66	88.1863	0.953	−0.302	55.0041	26.613
3	129	10	55.0041	26.613	75.8378	0.975	−0.219	74.518	22.231
4	140	15	74.518	22.231	65.88	0.993	−0.109	94.397	20.035
5	149	20	94.397	20.035	54.60	1.000	−0.0006	114.398	20.022
6	158	26	114.398	20.022	44.00	0.990	0.549	134.217	30.921
7	168	32	134.217	30.921	33.80	0.999	0.179	154.206	34.518
8	179	37	154.206	34.518	24.9[illegible]	0.995	0.099	174.112	36.510
9	188	34	174.112	36.510	14.11	0.984	−0.177	193.79	32.952
10	198	30	193.79	32.952	5.141	0.818	−0.574	210.168	21.467
11	209	27	210.168	21.467	5.65	−0.206	0.978	206.036	41.038
12	219	23	206.036	41.038	22.21				

Here, the distance between the percsuer and the bomber is less ~~greater~~ than 10km after 10 minutes of percsuing. So, the shooting will be abandoned. bomber will shoot.

Lecture-07 Date- 07/05/2023

3.3 Simulation of a Chemical Reaction

Letus consider a case where two chemicals Ch1 and Ch2 react to produce a third chemical Ch3. The rate of the reaction depends upon ratio of ch1 and ch2, the temperature, the humidity, the pressure etc.

In addition to the forward reaction, there may be also backward reaction, where ch3 decomposes back to Ch1 and Ch2.

If C_1, C_2 and C_3 are the amounts of Ch_1, Ch_2 & Ch_3 at any instant of time t, the increase rates of C_1, C_2 and C_3 are given by:

$$\frac{dC_1}{dt} = k_2C_3 - k_1C_1C_2$$

$$\frac{dC_2}{dt} = k_2C_3 - k_1C_1C_2 \qquad \text{where } k_1, k_2 \text{ are constants.}$$

$$\frac{dC_3}{dt} = 2k_1C_1C_2 - 2k_2C_3$$

The simulation of the reaction will determine the state of the system, that in the value of the quantities C_1, C_2 and C_3.

If $C_1(t)$, $C_2(t)$ and $C_3(t)$ are quantities of the three chemicals at time t, then at time $t+\Delta t$, the quantities are:

$$C_1(t+\Delta t) = C_1(t) + \frac{dC_1(t)}{dt}\Delta t$$

$$C_2(t+\Delta t) = C_2(t) + \frac{dC_2(t)}{dt}\Delta t$$

$$C_3(t+\Delta t) = C_3(t) + \frac{dC_3(t)}{dt}\Delta t$$

Taking $C_1(0)$, $C_2(0)$ and $C_3(0)$ as the quantities of Ch_1, Ch_2 and Ch_3, at time zero, that is when the reaction starts the state of the system at time Δt will be:

$$C_1(\Delta t) = C_1(0) + [k_2 C_3(0) - k_1 C_1(0) \cdot C_2(0)]\Delta t$$

$$C_2(\Delta t) = C_2(0) + [k_2 C_3(0) - k_1 C_1(0) C_2(0)]\Delta t$$

$$C_3(\Delta t) = C_3(0) + [2k_1 C_1(0) \cdot C_2(0) - 2k_2 C_3(0)]\Delta t$$

Now, using the state of the time $2\Delta t$, $3\Delta t$ and so on, will be computed. This will continue till the specified time of reaction T will be reached.

Chemical Reactor

$k_1 = 0.025$ $k_2 = 0.01$ time = 15.00

$C_1(0) = 50.0$ $C_2(0) = 25.0$ $C_3(0) = 0.0$

Time	C_1	C_2	C_3
0.00	50.00	25.00	0.00
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1.0			
1.1			
1.2			
1.3			
1.4			
1.5			
1.6			
1.7			

Chapter 04

Random Numbers

Random numbers
- → True
- → Pseudo

Properties :
i) uniformity
ii) independence

4.1, 4.2, 4.3, 4.4, 4.5, 4.6

Conguence Method or Residue Method

{mixed} → $r_{i+1} = (ar_i + b) \bmod m$

$a > b$
$b > 0$
$m > P$
$a, b, m \rightarrow$ constants

$r_1 = (1 \times 13 + 1) \bmod 19$

$= 14 \bmod 19$

$= 14$

$a = 13$
$b = 1$
$m = 19$
$r_0 = 1$ (Seed)

$r_2 = (14 \times 13 + 1) \bmod 19$

$= 183 \bmod 19$

$= 12$

. . . .

r_i & r_{i+1} are ith and i+1th rn.

if $a = 1$, then $r_{i+1} = (r_i + b) \bmod m$ → Additive type

if $b = 0$, then $r_{i+1} = ar_i \bmod m$ → Multiplicative type

Lecture-08 Date-21/05/2023

Combined Congruential Generator

$$r_i = \left(\sum_{j=1}^{k} (-1)^{j-1} \cdot r_{i,j} \right) \bmod m_1 - 1$$

with

$$R_i = \begin{cases} \frac{r_i}{m_i}, & r_i > 0 \\ \frac{m_i - 1}{m_i} & r_i = 0 \end{cases}$$

The maximum possible period for such generator is

$$P = \frac{(m_1 - 1)(m_2 - 1) \cdots (m_k - 1)}{2^{k} - 1}$$

Qualities of an efficient Random Number Generator

- Should have sufficiently long cycle
- Should be replicable
- Should fulfill the requirements of uniformity and independence.
- Should be fast and cost effective (time & space)
- Should be portable to different computers & ideally to different programming languages.

Testing Numbers for Randomness

A sequence of random numbers is considered to be random if,

i) The no.s are uniformly distributed

ii) The numbers are not serially auto-correlated.

Uniformity Test

Two tests available:

i) Kolmogorov – Smirnov Test

ii) Chi-Square Test

Example – 4.2: The Kolmogorov-Smirnov Test is to be performed to test the uniformity of following random numbers with a level of significance of $\alpha = 0.05$.

0.24, 0.89, 0.11, 0.61, 0.23, 0.86, 0.41, 0.64, 0.50, 0.65

Sol^n:

The calculation of the test is given below:

	i	1	2	3	4	5	6	7	8	9	10
	R_i	0.11	0.23	0.24	0.41	0.50	0.61	0.64	0.65	0.88	0.89
	i/N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
+) Derivation	$i/N - R_i$	—	—	0.06	—	00	~~0.06~~	0.06 ~~0.15~~	0.15 ~~0.04~~	0.04	0.11
-) Derivation	$R_i - \frac{(i-1)}{N}$	0.11	0.13	0.04	0.11	0.10	0.11	0.04	—	0.06	—

Maximum positive deviation = 0.15

" negative " = 0.13

Giving the largest deviation = 0.15

The critical value of deviation for $\alpha = 0.05$ and $N = 10$ is 0.410. which is ~~less~~ greater than 0.15.

Since $0.15 < 0.410$, the given random numbers are uniform at 95% level of significance.

$$\boxed{\begin{aligned} \alpha &= 100 - 95 \\ &= 05/100 \\ &= 0.05 \end{aligned}}$$

Chi-Squared Test

The Chi-squared test uses the simple statistic

$$\chi_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

O_i = Observed value

E_i = Expected value

Degree of freedom = $n-1$

$$E_i = \frac{N}{n}$$

N = total no. of observations

n = no. of classes

Example-4.3: The two-digit random numbers generated by multiplicative congruential method are given below. Determine Chi-square. Is it acceptable at 95% confidence level?

36, 91, 51, 02, - - - - 81, 26, 85

Soln:

The given 100 random numbers can be divided into 10 classes as given below: $E_i = \frac{N}{n} = \frac{100}{10} = 10$

Class	Count	Frequency (O_i)	Difference ($O_i - E_i$)	Difference^2 $(O_i - E_i)^2$
$0 < r \leq 10$		13	3	9
$10 < r \leq 20$		8	2	4
$20 < r \leq 30$		9	1	1
$30 < r \leq 40$		13	3	9
$40 < r \leq 50$		7	3	9
$50 < r \leq 60$		13	3	9
$60 < r \leq 70$		5	5	25
$70 < r \leq 80$		12	2	4
$80 < r \leq 90$		12	2	4
$90 < r \leq 100$		8	2	4
				78

$$\text{Chi-Square} = \frac{78}{10} = 7.8$$

For 9 (10-1) degrees of freedom, at 95% confidence level, the acceptable value of chi-square is 16.9, which is more than 7.8. Hence given set of random numbers is acceptable.

Lecture-09 Date-22/05/2023

Testing for Auto-Correlation

Example-4.4: Given below is a sequence of random numbers. Perform the chi-square tests to check the numbers for uniform distribution and serial auto-correlation.

07, 05, 96, 14, 10, - - - - -, 55, 18, 21.

Solution:

For checking uniformity, we will divide these in 10 class and take 90 rns to have expected value in each class as 9.

Class	Count	Frequency (O_i)	Difference ($O_i - E_i$)	$(O_i - E_i)^2$
$0 \leq R \leq 10$		9	0	0
$11 \leq R \leq 20$		9	0	0
$21 \leq R \leq 30$		12	3	9
$31 \leq R \leq 40$		8	1	1
$41 \leq R \leq 50$		12	3	9
$51 \leq R \leq 60$		9	0	0
$61 \leq R \leq 70$		8	1	1
$71 \leq R \leq 80$		8	1	1
$81 \leq R \leq 90$		8	1	1
$91 \leq R \leq 100$		7	2	4
				26

$$\text{Chi-square} = \frac{26}{9} = 2.9 < 16.9.$$

Since our value for 9 degree of freedom is well within the acceptable limit at 95% confidence level. So, the random numbers are uniformly distributed.

For the test of auto correlation, the overlapping pairs of random numbers are to be taken. Taking three classes, 0–33; 34–67, 68–100 for each random number in the pair, the number of pairs in each class is obtained as follows. Since there are 9 classes, the expected value is 10.

R_1	R_2	Count	Frequency (O_i)	$(O_i - E_i)$	$(O_i - E_i)^2$
≤ 33	≤ 33	~~**********~~	10	0	0
≤ 67	≤ 33	~~**********~~	10	0	0
≤ 100	≤ 33	~~*** *** ***~~	9	1	1
≤ 33	≤ 67	~~*******~~	7	3	9
≤ 67	≤ 67	~~********************~~	20	10	100
≤ 100	≤ 67	~~***********~~	11	1	1
≤ 33	≤ 100	~~*************~~	13	3	9
≤ 67	≤ 100	~~**** **~~	6	4	16
≤ 100	≤ 100	~~****~~	4	6	36
					172

Chi-square $(\chi_0^2) = \frac{172}{10} = 17.2$

The criterion value for χ^2 for 7 degrees of freedom at 95% confidence level is 14.1, which is less than 17.2. Thus the given sequence of random numbers is serially auto-correlated at 95% confidence level.

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Poker Test

- Card game
- test randomness
- 5 digit random number

71549 → five different digits

55137 → ~~two pair~~ apair

33669 → two pairs

55513 → three of a kind

44477 → Full House

77774 → four of a kind

88888 → five of a kind

Example-4.5: A sequence of 10,000 five digit random numbers has been generated. An ~~at~~ analysis of numbers indicate that, there are 3075 numbers having five different digits, 4935 having a pair, 1135 having two pairs, 695 having three of a kind, 105 having full house, 54 having four of a kind and one having five of a kind. Use poker test to determine if these random numbers are independent at $\alpha = 0.01$.

Solution: The calculations of χ^2 are given below:

Combination Distribution (i)	Observed Distributions (O_i)	Expected (E_i)	$\frac{(O_i-E_i)^2}{E_i}$
Five different digits	3075	3024	0.8601
Pair	4935	5040	~~0~~ 2.1875
Two-pairs	1135	1080	1.8750
Three of a Kind	695	720	0.868
Full house	105	90	2.5
Four of a kind	54	45	1.8
Five of a kind	01	01	0.0
	10,000	10,000	10.0907

The critical value of χ^2 for 6 degrees of freedom at $\alpha = 0.01$ is 16.8. The value of chi-square obtained from calculation is 10.097 which is less than 16.8. Hence, these random numbers are independent.

Ex - 4.10, 4.11.

Lecture-10 Date- 24/05/2023

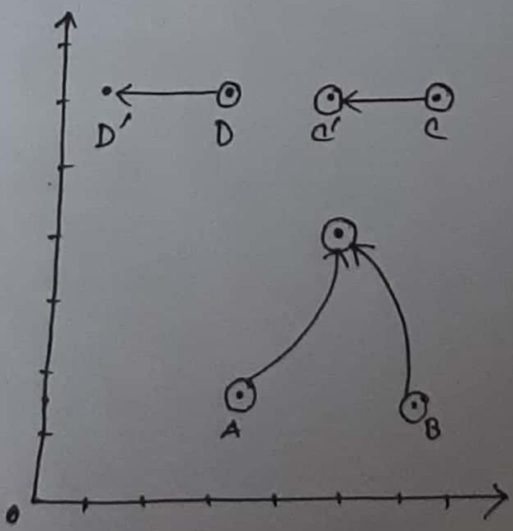
Chapter 03

3.6 A serial chase Problem

- moving objects থাকবে যা একটি আরেকটিকে chase করবে। যখন একটি তার target কে hit করবে তখন chase stop হবে।

Applications

- Applied mathematics, applied chemistry
- fluid
- Chemical reaction



$V_a = 35$ km/hr

$V_b = 25$ km/hr

$V_c = 15$ km/hr

$V_d = 10$ km/hr

* D কে 1st randomly move করা হবে।
* pure pursuit দিয়ে solve A, B এর distance
* Distance 0.005 হলে chase ends.

Lecture-01 (MAM) Date-02/09/2023

Chapter-05

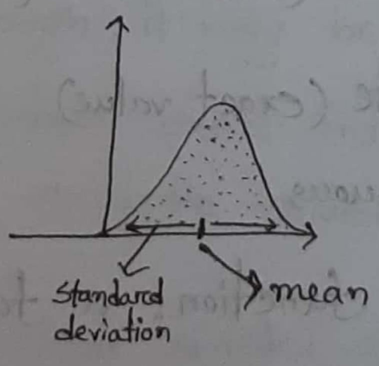
Distribution Function

Cumulative Distribution Function: It is a function which gives the probability of a random variable being less than or equal to a given value. Denoted by $P(x_i)$ which implies that x takes the values less than or equals to x_i.

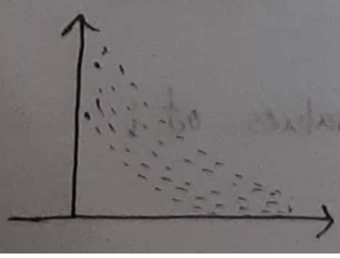
* Distribution of data
* Data structures; function for the structure

Data Properties:

i) Center limit theorem: center-এ combined থাকে



ii) Skewed: creates assymetrical, skewed curve



Examples of Distribution function –

i) Gamma distribution

ii) Gausian "

iii) Poison "

** Applications of Distribution

Algorithm → 2 types :

1. Direct input to output → Deterministic

2. বার বার randomly run করলে → Non-deterministic

System → 2 Types :

1. Deterministic

2. Non-deterministic (Stochastic – যে randomness maintain করে)

↳ i) Discrete (exact value)

ii) Continuous

Discrete probability function : a function that can take a discrete number of values (not necessarily finite).

Properties :

i) $P(x_i) \geq 0$ for all values of i

ii) $\sum^{n} P(x_i) = 1$

Lecture-02 Date - 11/05/2023

Chapter 01

Basic Simulation Modeling

* The Nature of Simulation

Assumption → usually take the form of mathematical or logical relationships., constitute a model that is used to try to gain some understanding of how the system behaves.

$y = mx + c$

This can be a model. Because there is an assumption or mapping between the dependent and independent variable. and theres a relation between them.

Analytical solution : If the relationships that compose the model are simple enough it may be possible to use mathematical methods to obtain exact information on questions of interest; this is called analytic solution.

যখন system too complex → simulation

** Applications of Simulation

** Problems or impediments of simulation:

1. Models used to study large-scale systems tend to be very complex and writing computer programs to execute them can be a difficult task.

⇒ This problem has been addressed by the development of excellent software products that automatically provide many of the features needed to program a simulation model.

2. Large ~~#~~ amount of computer time is required sometimes

⇒ This difficulty has become much ~~cheaper~~ less severe as computers become faster and cheaper.

3. There appears to be an unfortunate impression that simulation is just an exercise in computer programming that too a complicated one.

⇒ Consequently, many simulation studies have been composed of heuristic model building, programming and a single run of the program to obtain the answer.

Need design and analysis of simulation models, methodology carefully.

* Systems, Models and Simulation

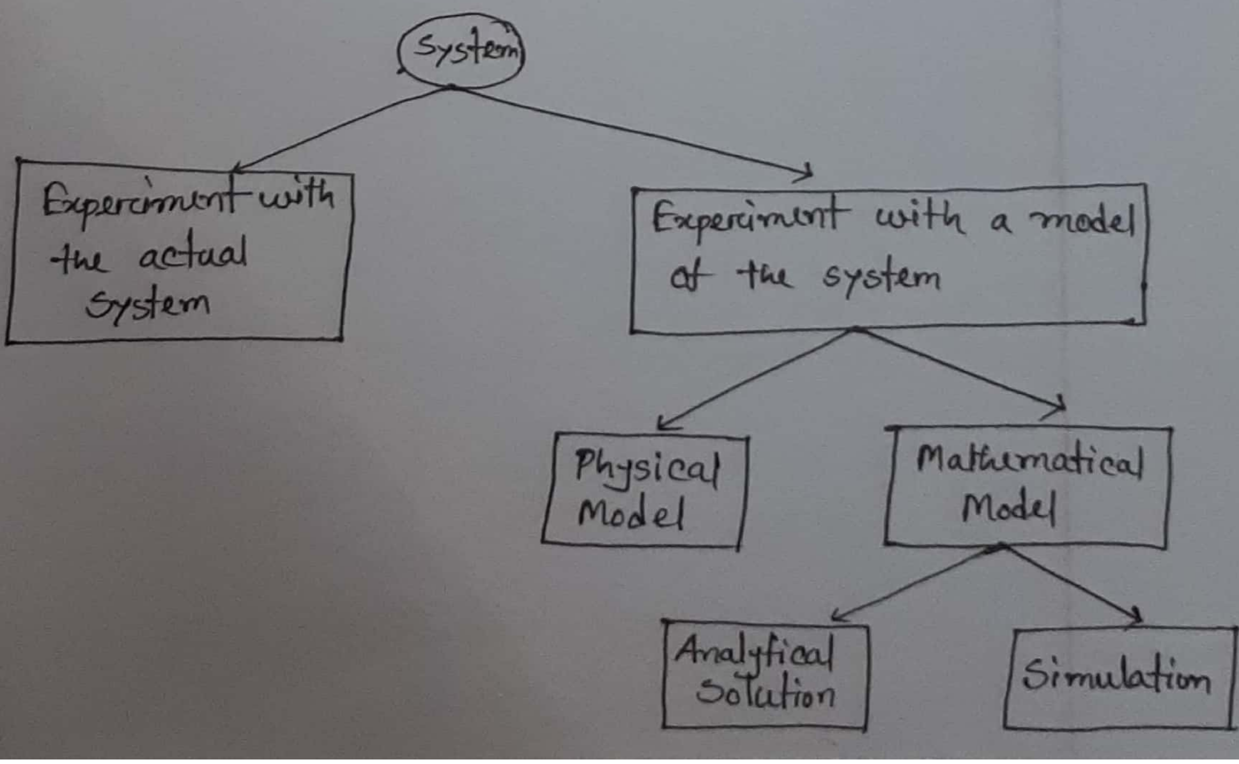
System - is defined to be a collection of entities that act and interact together toward the accomplishment of some logical end.

Two types -

I) Discrete - variable changes instantaneously at a point of time. Example - bank

II) Continuously - state variable changes continuously with time. Example - Airplane.

** Ways to study a System



Lecture - 03 Date - 16/05/2023

* Discrete - Event Simulation :

Example 1.1

** Time - Advance mechanism

Example 1.2

** Components and Organization of a Discrete - event simulation model.

* Simulation of a Single-server queing system

Problem statement

Intuitive Explanation

Lecture-04

Date- 24/05/2023

Chapter 04

Review of Basic Probability & Statistics

* Why probability and statistics are needed?

⇒ Probability and statistics are needed to understand how to model probablistic system, validate the simulation model, choose the input probability distribution, generate random number samples from these distributions, perform statistical analysis of the simulation output data and design the simulation experiments.

** random variables and their properties

Experiment – a process whose outcome is not certain

Random variable – is a function or a rule that assign a real no. to each point in the sample space.

Distribution function –

$$F(x) = P(X \le x) \quad \text{for } -\infty < x < \infty$$

Properties

Types –

i) Discrete distribution function (countable no.)

ii) Continuous distribution function

Probability mass function - for discrete random ~~number~~ variable.

Probability density function - for continuous random variable.

mean $\beta \rightarrow$ parameter

Joint Probability Distribution

Mean $(\mu) \rightarrow$ expected value

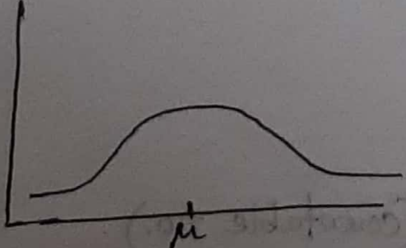
* Why mean/median is needed?

⇒ because exactly statistics কে represent করে

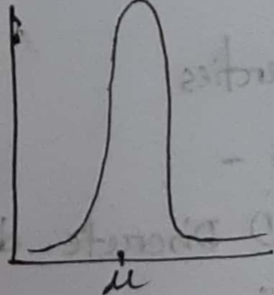
Variance - deviation (data থেকে কত দূরে আছে)

$$\sigma_i^2 = E[(x_i - \mu_i)^2] = E(x_i)^2 - \mu_i^2$$

Gaussian Distribution - bell shaped.



Larger variance



Smaller variance

Variance Properties

* Variance is used for standard deviation
  - What type of data ⇒ continuous
  - Dimension of data ⇒ one dimensional

Co-variance
  - are symmetrical
  - 2 dimensional i.e 2x2 matrix
  - diagonal values σ^2

Correlation

$$\rho_{ij} = \frac{c_{ij}}{\sqrt{\sigma_i^2 \sigma_j^2}}$$

Lecture-05 Date- 28/05/2023

* Simulation Output Data

Stochastic process - a collection of similar random variable ordered over time

Discrete time stochastic process → uses discrete random variables.

Continuous " " " → uses continuous random variables.

Example - 4.19, 4.20

Co-variance stationary (over time এ তার probability-র output pattern change হবে না; means constant)

- certain properties

- $\left.\begin{array}{l}\mu_i = \mu \\ \sigma_i^2 = \sigma_i\end{array}\right\}$ mean & variance change হবে না

- multiple dimension (more than 1)

** Interarrival time -

Service time -

Interarrival time + service time → input

Delay time → output

$\rho = 5$ বলা যাবে না, cause $\rho < 1$

$$\rho = \frac{\lambda}{\omega} < 1$$

λ = arrival rate at unit time

ω = service time

$\rho = \frac{\lambda}{\omega} = 1$ means, যারা এসেছে সবাইকেই service দেয়া হয়েছে।

Correlation

Figure 4.10

* Estimation of means, variance and correlations

$$\bar{X}(n) = \frac{\sum_{i=1}^{\infty} x_i}{n}$$

unbiased হওয়ার কারণে expected, $E[\bar{X}(n)] = \mu$

Figure 4.12, 4.13

* Confidence intervals and hypothesis tests for mean

hypothesis - assumption

Central limit theorem - central tendency (mean এর কাছাকাছি সব data এসে জমা হচ্ছে)

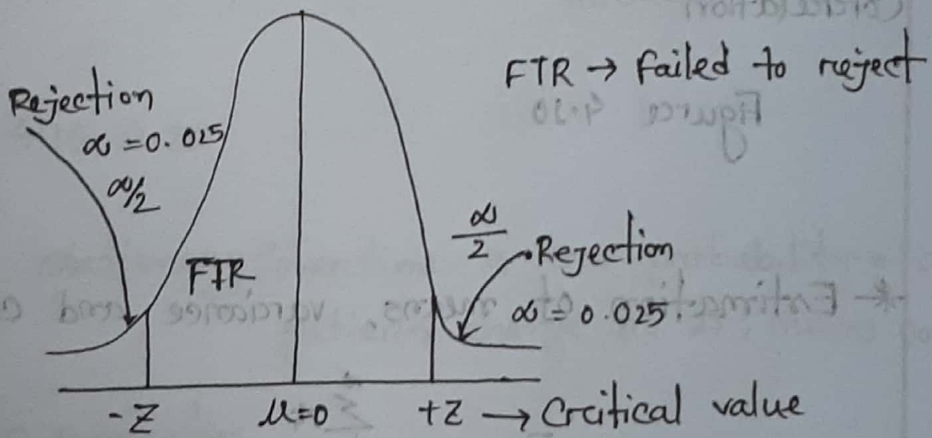
*Theorem 4.1

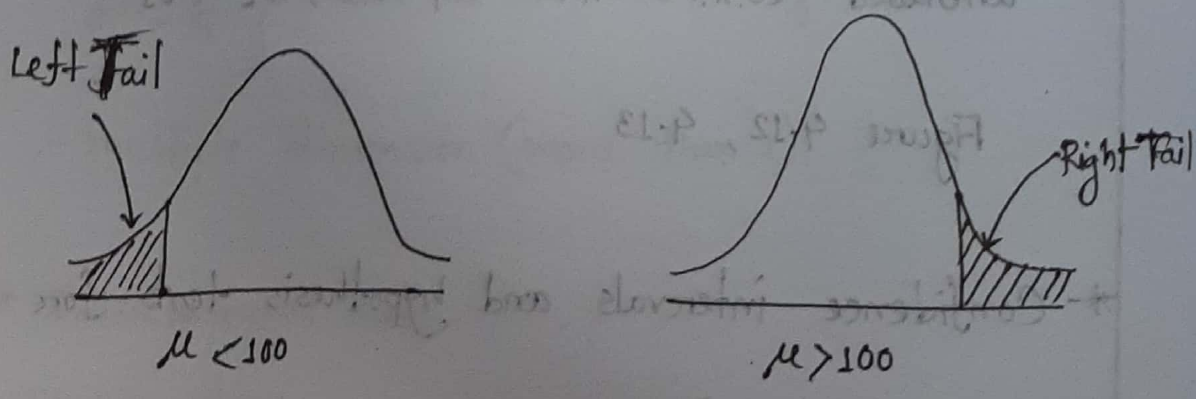
mean = 0 ; means one positive, another side negative

$\mu = 0$, $\sigma^2 = 1$] → standard normal random variable

$\phi(z)$ → Standard normal random variable function

Fig - 4.15





(a) (b)

Hypothesis:

i) Null Hypothesis (H_0) : Ex – $\mu = 0$

ii) Alternate Hypothesis (H_a) : Ex – $\mu \neq 100$

Problem statement

Solution → test statistics; value-র সাথে compare করে

$C = 0.95$ means 95% confidence level

α = significant level

$\alpha = 0.05$

Z_c compute করে

T-dist → special case of normal distribution

Rules → যা define করে

i) No. of sample, n এর value < 30 হলে

population standard deviation unknown হলে → T-distribution

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \qquad n < 30, \quad \theta = \text{unknown}$$

ii) $n > 30$ → normal distribution

$Z_c =$

Example - 4.27

2 Types error

Table 4.2

i) $\alpha \rightarrow$ if one reject H_0 when infact its true

ii) $\beta \rightarrow$ if one failed to reject, but it was false

Example 4.29

The Strong Law of large number

$$\bar{X}(n) \rightarrow \mu \text{ w.p. } 1 \text{ as } n \rightarrow \infty$$

Theorem 4.2

Example - 4.31

Figure - 4.~~33~~18

Lecture-6 (Online) Date- 13/06/2023

Chapter 06

Selecting Input Probability Distribution

6.2 Useful ~~Distribution~~ Probability Distribution

Parameterize → to define

Parameterization of Continuous Distribution

If the parameters are defined correctly, the probability ~~distribution function~~ density function can be classified, on the basis of their physical or geometric interpretation, as being one of three basic types:

1. Location parameter (γ): specifies an abscissa (x-axis) location point of a distribution's range values. Usually γ is the mid point (μ for normal distribution) or lower endpoint of the distributions range.

It is also called shift parameters as γ changes, the associated distribution merely shifts left or right without otherwise changing.

$$\text{r.v } X,\ \gamma = 0$$

$$\text{Then r.v } Y = X + \gamma$$

Y has a location parameter γ.

CamScanner

2. Scale parameter (β): determines the scale (or unit) of measurement of the values in range of the distribution. The standard deviation σ is a scale parameter for the normal distribution.

A change in β compresses or expands the associated distribution without altering its basic form.

r.v X, $\beta = 1$

Then r.v $Y = \beta X$,

Y has scale parameter of β.

3. Shape parameter (α): determines the basic form or shape of a distribution within the general family of distributions of interest.

A change in α generally alters a distribution's properties more fundamentally than a location or ~~sh~~ scale.

Some distributions doesn't have a shape parameter like exponential and normal.

while others like beta may have two.

From Table 6.3

i) Uniform Distribution

ii) Exponential Distribution

iii) Normal Distribution.

Chapter 07

Random Number Generators

Random variates – random variable to value; particular outcome of a random variable.

7.2 Linear Congruential Generator (LCG)

A sequence of integers $Z_1, Z_2 \ldots$ is defined by the recursive formula

$$Z_i = (aZ_{i-1} + c) \bmod m$$

where m = the modulus
a = the multiplier
c = the increment
Z_0 = the seed or starting value.

non-negative integers

should satisfy $0 < m$
$a < m$
$c < m$
$Z_0 < m$

Example - 7.2 : LCG defined by $m = 16$, $a = 5$, $c = 3$, $Z_0 = 7$.

Soln:

$$Z_i = (5 Z_{i-1} + 3) \bmod 16 \qquad U_i = \frac{Z_i}{m}$$

i	Z_i	U_i
0	7	–
1	6	0.375
2	1	0.062
3	8	0.500
4	11	0.6875
5	10	0.625
6	5	0.312
7	12	0.750
8	15	0.9375
9	14	0.875
10	9	0.562
11	0	0
12	3	0.187
13	2	0.125
14	13	0.812
15	4	0.250
16	7	0.437
17	6	0.375

Chapter 08

Generating Random Variates

8.2 General Approaches to Generate Random ~~variable~~ Variates:

Inverse Transform

Suppose that we wish to generate a random variate X that is continuous and has distribution function F that is continuous and strictly increasing when $0 < F(x) < 1$.

Let F^{-1} denote the inverse of the function F. Then an algorithm for generating a random variate X having distribution function F is as follows:

1. Generate $U \sim U(0,1)$
2. Return $X = F^{-1}(U)$

$F^{-1}(U)$ will always be defined, since $0 \leq U \leq 1$ and the range of F is $[0,1]$.

The illustration of this algorithm graphically is given here where the random variable corresponding to this distribution function can take on either positive or negative values.

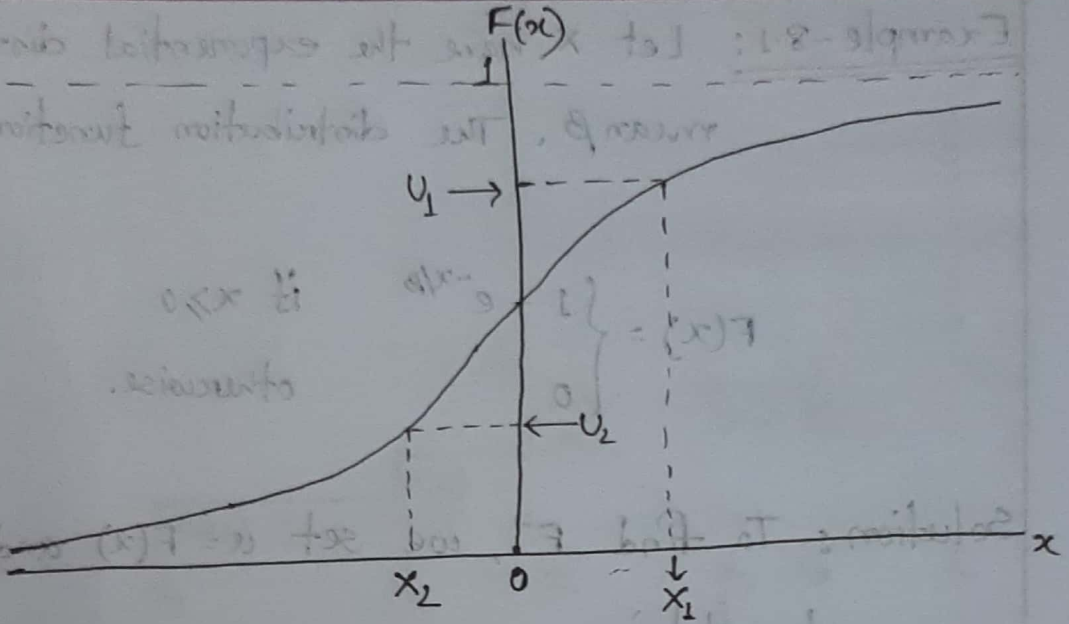


Figure: Inverse transform method for continuous random variables.

In the figure, random number U_1 results in the positive random variate X_1 and the random number U_2 results in the negative random variate X_2.

For any ~~random~~ real number x, $P(X \leq x) = F(x)$.

Since F is invertible, we have

$$P(X \leq x) = P(F^{-1}(U) \leq x)$$
$$= P(U \leq F(x))$$
$$= F(x)$$

where the last equality follows since $U \sim U(0,1)$ and $0 \leq F(x) \leq 1$.

Example-8.1: Let X have the exponential distribution with mean β. The distribution function is

$$F(x) = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Solution: To find F^{-1}, we set $u = F(x)$ and solve for x to obtain

$$F^{-1}(u) = -\beta \ln(1-u)$$

Thus to generate the desired random variate, we first generate a $U \sim U(0,1)$ and then let $X = -\beta \ln U$.

** When X is discrete, the distribution function is,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i)$$

where $P(x_i)$ is the probability mass function

$$P(x_i) = P(X = x_i)$$

The algorithm is then as follows:

1. Generate $U \sim U(0,1)$
2. Determine the smallest positive integer I such that $U \leq F(x_I)$ and return $X = x_I$

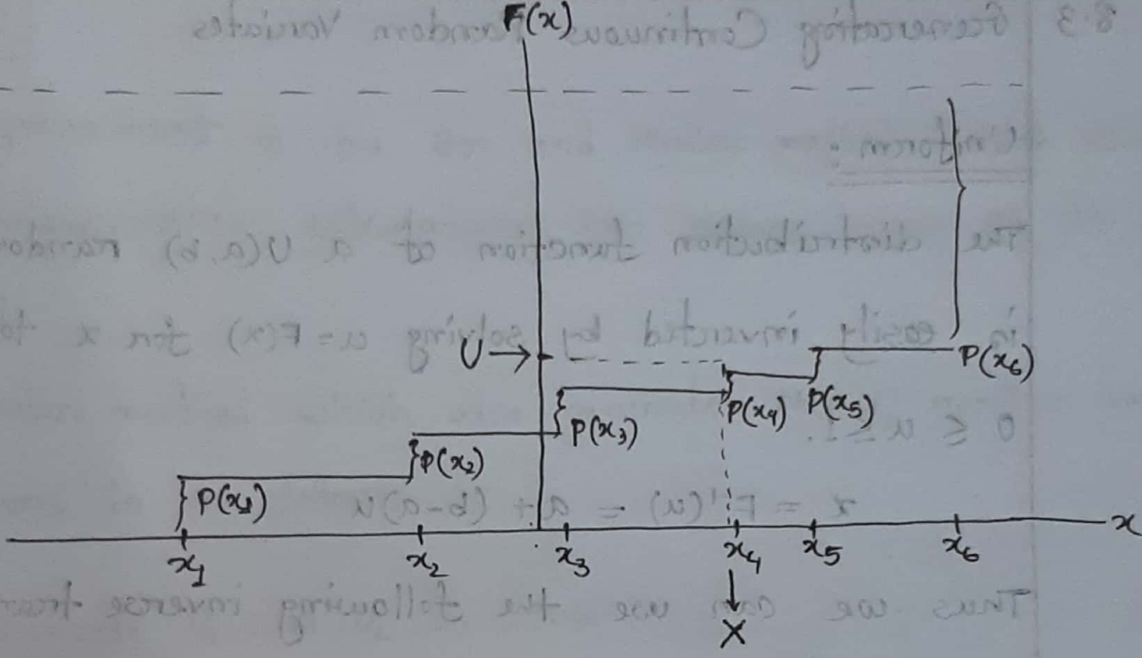


Figure: Inverse-transform method for discrete random variable.

8.3 Generating Continuous Random Variates

Uniform:

The distribution function of a $U(a,b)$ random variable is easily inverted by solving $u = F(x)$ for x to obtain for $0 \le u \le 1$.

$$x = F^{-1}(u) = a + (b-a)u$$

Thus we can use the following inverse transform method to generate X:

1. Generate $U \sim U(0,1)$
2. Return $X = a + (b-a)U$

Exponential:

The exponential random variable with mean $\beta > 0$, we use the following inverse transform algorithm to generate X:

1. Generate $U \sim U(0,1)$
2. Return $X = -\beta \ln U$

In order to make the correlation positive, U is used instead of $1-U$.

Normal:

An improvement to the Box and Muller method, which eliminates the trigonometric calculations has become known as the polar method.

The polar method which also generates $N(0,1)$ random variates in pairs is as follows:

1. Generate U_1 and U_2 as IID $U(0,1)$; Let $V_i = 2U_i - 1$ for $i = 1, 2$; and let $W = V_1^2 + V_2^2$

2. If $W > 1$, go back to step 1. Otherwise, let $Y = \sqrt{(-2\ln W)/W}$, $X_1 = V_1 Y$ and $X_2 = V_2 Y$. Then X_1 and X_2 are IID $N(0,1)$ random variates.

Since a rejection of U_1 and U_2 can occur in step 2, the polar method will require a random number of $U(0,1)$ random variates to generate each pair of $N(0,1)$ random random variates.