Semister Final → 20-21

Simulation and Modeling, CIT-323

Mahabub sir → 3 set (chap - 1, 2, 3, 4)

Book → by D. S. HIRA

Masud sir → 3 set (David kelton)

set 1 → chapter - 1 (Basic simulation modeling)

set 2 → chap - 4 (Review of Basic probabity and statics)

chap - 5 (Building valid, credible, and Appropiriately Detaited Models)

Set 3 → chap - 6 (selecting input probability distribution)

Chap - 7 (Random number generator)

Chap - 8 (Generating Random variates)

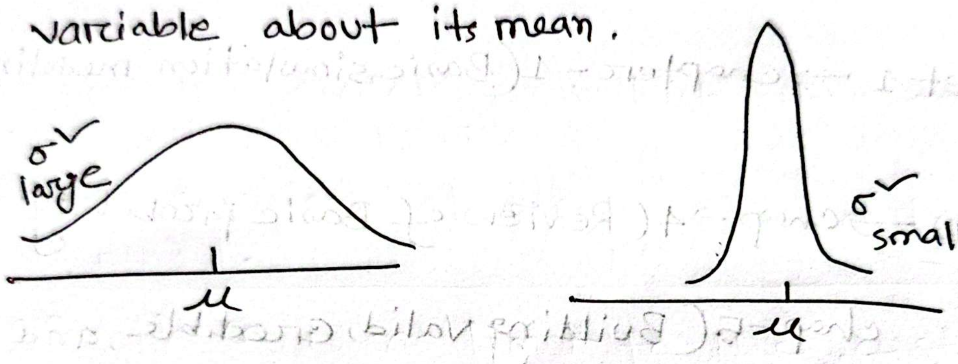
Chap - 09 (Output Data analysis for a single system.

Chapter 4 and 5 → 1 set

* How to compute variance, covariance, and correlation for simulation study.

Variance

The variance is a measure of the dispersion of a random variable about its mean.



$$\sigma^2 = E(x_i^2) - \mu_i^2$$

Co-Variance

The co-variance is the measure of dependence between two random variables. If the two random varibles are X_i and X_j.

Then the co-variance,

$$C_{ij} = E(X_i X_j) - \mu_i \mu_j$$

Correlation

Correlation is a statistical measure that describes the relationship between two random variables.

The correlation ρ_{ij} is defined by,

$$\rho_{ij} = \frac{c_{ij}}{\sqrt{\sigma_i^2 \sigma_j^2}}$$

* why stochastic process is used to simulate the output data

A stochastic process is used to simulate situations where outcomes are uncertain or random, it helps predict possible ~~outcome~~ results when we can't know for sure what will happen, but we can estimate based on probabilities.

যদি কম মার্ক এ আসে তাহলে example বাদ

Examples: Imagine you are trying to guess how many customers visits a shop each day. Somedays there are 20, other days 50, and it changes randomly. A stochastic process helps simulate these changes, giving you a range a possible outcomes instead of just one fixed number.

* Illustrate correlation function of the process of delays in queue $D_1, D_2 \ldots$ for the M/M/1 queue.

The M/M/1 model is a queueing process in which customer arrive at one server and wait in a queue ~~un~~ until the server in avaiable customers are serviced in the order in which they arrived. The server services at most one customer at a time. There is no limit to the ~~customer~~ number of customer who can wait in the queue.

Consider the output process D_1, D_2 for a covariance stationary M/M/1 queue with $\rho = \lambda/\omega < 1$.

Here λ = the arrival rate

ω = Service rate

From the results in Delay (1968), one can compute ρ_j, which we plot in the figure for $\rho = 0.5$ and 0.9

The correlation ρ_j are positive and monotonically decrease to zero as j increase. In particular, $\rho_1 = 0.99$ for $\rho = 0.9$ and $\rho_1 = 0.78$ for $\rho = 0.5$. The convergence of ρ_j to zero is considerably slower for $\rho = 0.9$

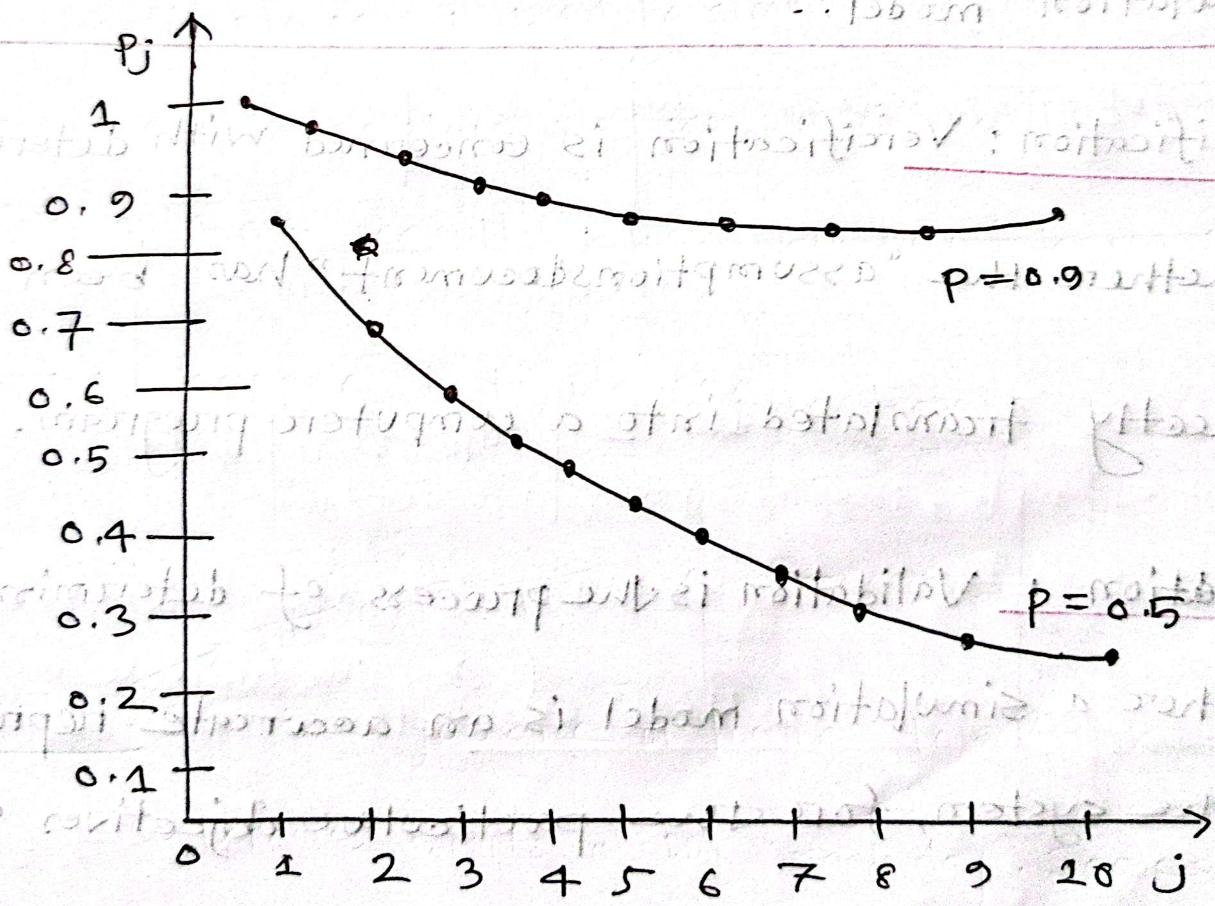


Figure: Correlation function ρ_j of the process $D_1, D_2, \cdots$ for the M/M/1 queue.

* What is random variable?

A random variable is a function (or rule) that assigns a ~~rela~~ real number (any number greater than $-\infty$ and less than ∞) to each point in the simple space S.

* Define verification, validation and credibility in a simulation model.

Verification: Verification is concerned with determining whether the "assumptions document" has been correctly translated into a computer program.

Validation: Validation is the process of determining whether a simulation model is an accurate representation of the system, for the particular objectives of the study.

Credibility: Credibility refers to how trustworthy and believable the simulation results are.

* Mention the issues in an accreditation decision. (1)

1. Verification and validation that have been done.
2. Credibility of the model.
3. Simulation model development and use history.
4. Quality of the data that are available.
5. Quality of the documentation.
6. know problems or limilation with simulation model.

* Show the timing relationship between validation, verification and establishing credibility.

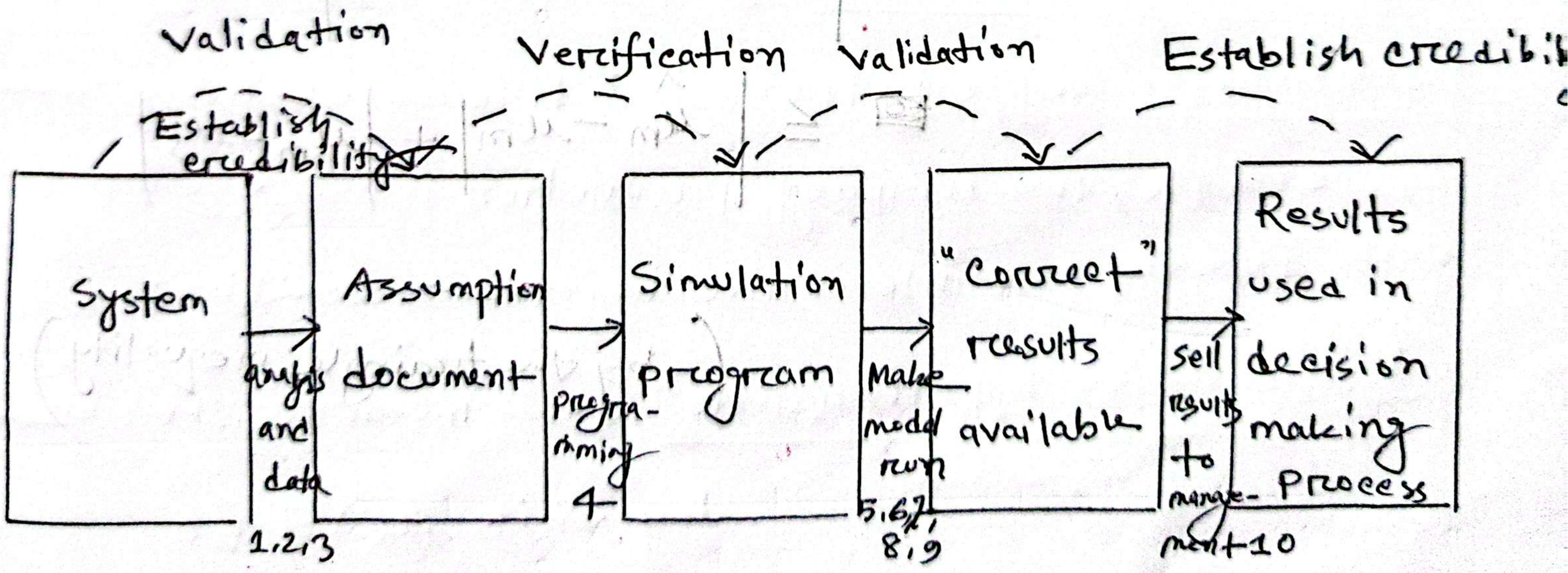


Figure: Timing relationship of validation, verification and establishing credibility.

* Derive the mathematical formulation in a good estimate of the mean of the system using both validation and output analysis (2).

suppose, we want to estimate the mean μ_s of some system. We construct a simulation model whose corresponding mean is μ_m. We make a simulation run and obtain an estimate $\hat{\mu}_m$ of μ_m.

$$\therefore \text{Error in } \hat{\mu}_M = \left|\hat{\mu}_M - \mu_s\right|$$

$$= \left|\hat{\mu}_m - \mu_m + \mu_m - \mu_s\right|$$

$$\leq \left|\hat{\mu}_m - \mu_m\right| + \left|\mu_m - \mu_s\right|$$

(by the traingle inequality)

chap-04

* Explain hypothesis testing including null and alternative hypothesis for mean. (2)

Null hypothesis (H_0):

There is no effect in the population. It is the starting assumption in staties. It says there is no relationship between groups. For a example, A company claim its average production is 50 units per day, then;

Null hypothesis (H_0): The mean number of daily visits $(\mu) = 50$.

Alternative hypothesis (H_1): There is an effect in the population. It suggests there is a difference between group. For example, the company production is not equal to 50 units per day then alternative hypothesis would be;

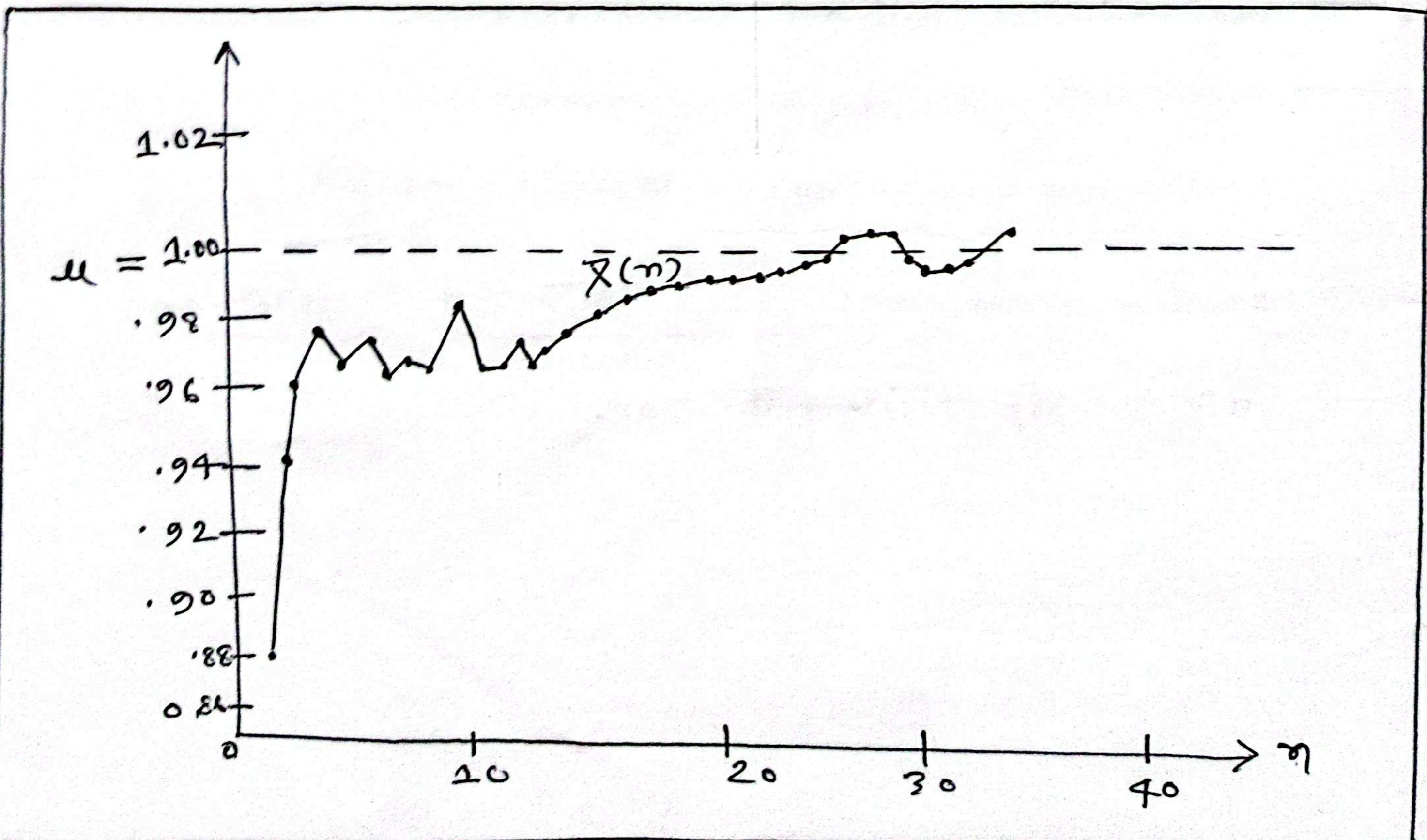
Alternative hypothesis (H_1): The mean number of day visits $(\mu) \neq 50$

* What is the difference between type I error and type II error. (2)

Type I	Type II
If one rejects H_0 when in fact it is true, this is called Type I error,	2) If one fails to reject H_0 when it is false, this is called a Type II error.
False positive	3) False negative
symbol: α (alpha)	3) symbol: β (beta)

* Explain and illustrate the strong law of large number.

Let $X_1, X_2, \dots$ are IID random variables with $\mu = 1$ and $\sigma^2 = 0.01$. In this below figure the values of $\bar{X}(n)$ for various n that resulted from sampling from this distribution. Note that $\bar{X}(n)$ differed from μ by less than 1 percent for $n \geq 28$.



$\bar{X}(n)$ for various values of n when the X_i are normal random variat

* Explain the inspection approach of statistical procedure for comparing real-world observation and simulation output analysis. (3)

For a real-world system of interest is the M/M/1 queue with $p = 0.6$ and that the corresponding simulation model is the M/M/1 queue with $p = 0.5$, in both cases arrival rate is 1 and the output process of interest is $D_1, D_2, \ldots$

Let, $X = \frac{\sum_{i=1}^{200} D_i}{\;} \rightarrow$ for the system

$Y = \frac{\sum_{i=1}^{200} D_i}{200} \rightarrow$ for the model.

We shall attempt to determine how good a representation the model is for the system for comparing an estimate for $\mu_Y = E(Y) = 0.49$. with an estimate $\mu_X = E(X) = 0.87$

Experiment	$\hat{\mu}_X$	$\hat{\mu}_Y$	$\hat{\mu}_X - \hat{\mu}_Y$
1	0.90	0.70	0.20
2	0.70	0.71	-0.01
3	1.08	0.35	0.73

Table is the results for three experiment with the inspection approach.

The table gives the results of three independent simulation experiments, each corresponding to a possible application of the inspection approach.

chap-6

* List some source of randomness ~~of~~ for common simulation application.

Type of system	Source of randomness
Manufacturing	Processing times, machine times to failure, machine repair times
Defense-related	Arrival times and payloads of missiles or airplanes, outcome of an engagement, miss distance for munitors.
communications	Interarrival times of messages, message type, message length.
Transportation	Ship-loading times, interarrival times of customers to a subway.

→chap-6

* Write the properties of Location, scale and shape parameters of continous distributions.

A Location parameters.

A location parameter γ specifies an abscissa (x axis) location point of a distribution range of values; usually γ is the midpoint or lower endpoint of the distribution range. As γ changes, the associated distribution merely shift left or right without otherwise changing.

A scale parameter

A scale parameter β determine the scale of measurement of the values in the range of the distribution. A changes in β compress or expands the associated distribution without altering its basic form.

A shape parameter

A shape parameter α determines the distinct from location and scale, the basic form or shape of a distribution within the general family of distribution of interest.

$\rightarrow$ chap-8

* What is inverse transform function?

The Inverse transform method is a technique for generating random variables from any continous probability distribution using uniform random numbers.

$\rightarrow$ chap-8

* Let X have the exponential distribution with mean β, Generate the desire random variate using the inverse transform function.

The distribution function is;

$$F(x) = \begin{cases} 1-e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

So to find F^{-1}, we set $u = F(x)$ and solve for x to obtain

$$F^{-1}(u) = -\beta \ln(1-u)$$

Thus, to generate the desired random variate, we first generate a $U \sim U(0,1)$ and then let $X = -\beta \ln U$. [it is possible in this case to use U instead of $1-U$, since $1-U$ and U have the same $U(0,1)$ distribution].

CamScanner

chap-09

* What is the necessity of statistical analysis on simulation output data?

1. Quantifies Uncertainty – Measures variation due to randomness.

2. Supports Decision Making – Provides reliable insights for choosing the best option.

3. Enable comparisions – Objectively compares different scenarios or systems.

chp-7

* Many random number generators in use today are linear congruential generators, LCGs. Explain how to obtain the desired random numbers from LCGs.

Firstly we start a seed value Z_0.

Generate the next value Z_i using the recursive formula:

$$Z_i = (aZ_{i-1} + c) \pmod m$$

Where, m = modulus

a = multiplier

c = increment

Z_0 = initial seed

Now, convert each Z_i to a uniform random number $U_i \in [0, 1)$ by;

$$U_i = \frac{Z_i}{m}$$

~~The~~ This process produces pseudo-random numbers U_i that aim to behave like independent $U(0,1)$ values.

Group-4

* Describe different types of simulation models.

1. Discrete-Event simulation - changes occur at specific events

2. Continous simulation - System changes continously over time.

3. Monte carlo simulation - Uses random sampling to estimate outcomes.

4. Agent-based simulation - Models individual agents and their interactions.

5. Hybrid simulation - Combines multiple types.

* Describe and illustrate transient and steady-state behavior of a stochastic process

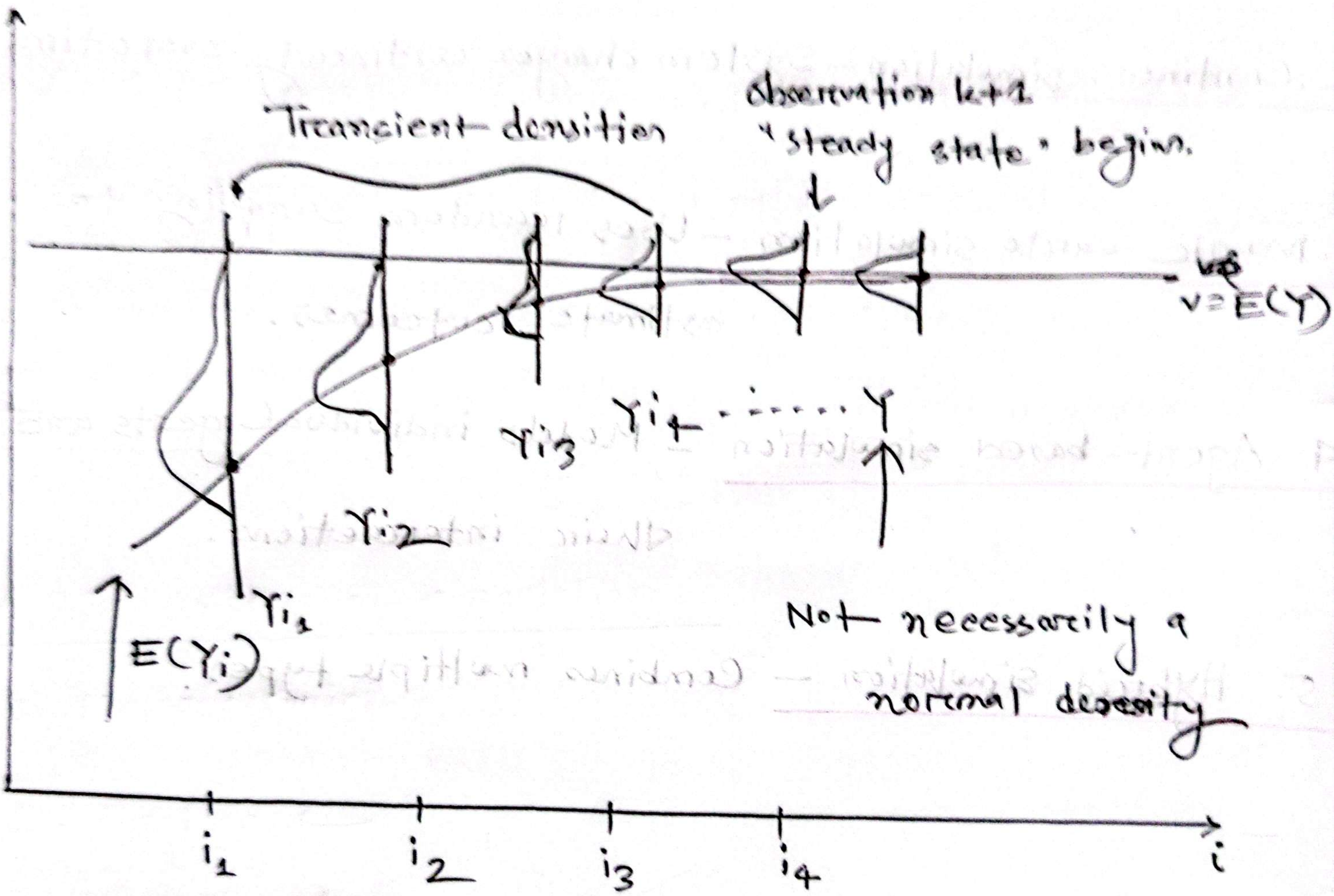


Figure: Transient and steady-state density function for a particular stochastic process $Y_1, Y_2 \ldots$ and initial condition I.

For fixed y and I, the probabilities $F_1(y/I), F_2(y/I) \ldots$ are just a sequence of number. If $F_i(y/I) \rightarrow F(y)$ as $i \rightarrow \infty$ for all y and for any initial conditions I, then $F(y)$ is called the steady-state distribution of the output process $Y_1, Y_2 \ldots$ Strictly speaking, the steady-state distribution $F(y)$ is only obtained in the limit as $i \rightarrow \infty$

* What are the advantages of simulation? Describes the applied area of simulation. (2)

Advantages of simulation:

1. Analyzes complex systems without real-world risks.
2. Saves time and cost by testing virtually.
3. Visualizes system behavior over time.
4.

Applied area of simulation

1. Manufacturing
2. Healthcare
3. Transportation
4. Finance
5. Military and Defense.
6. Environment.