Sub.: Chapter – 4

SAT SUN MON TUE WED THU FRI

DATE: / /

Example – 4.5 :

Total option for 5 digits = $10 \times 10 \times 10 \times 10 \times 10$

$= 100,000$

(1) For different 5 digits:

(*) $10 \times 9 \times 8 \times 7 \times 6 = 30,240$ টি

(*) $E_c = \frac{30,240}{100,000} \times 100,000 = 3024$

(2) One Pair: (55137)

(*) কোন ডিজিটটি জোড় হবে? ১০টির মধ্যে যেকোনো ১টি $\binom{10}{1}$

(*) ৫ টি জায়গার মধ্যে কোনো ২টি জায়গায় সেই জোড় বসাবে। $\binom{5}{2}$ উপায়ে।

(*) বাকি জায়গায় অন্য ৩ টি বসাবে।

So, $10 \times 10 \times 9 \times 8 \times 7 = 50,400$

$E_c = \frac{50,400}{100,000} \times 10000 = 5040$

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

③ Two Pair: (aabbc)

→ 10টি থেকে 2টি বেছে নেও (2 জোড় করার জন্য): $\binom{10}{2} = 45$

→ 5 টি জাতীয় ভিতর জোড়া 2টির জন্য 4টি জাতীয় বেছে নেও: $\binom{5}{2}\binom{3}{2} = 10 \times 3 = 30$

→ বাকি 2 জায়গায় জন্য 8টি অংকের 1টি যোগানো: $\binom{8}{1} = 8$

So, $45 \times 30 \times 8 = 10,800$

So, $E_e = 1080$

④ Three of kind: (aaabc)

→ 10টি থেকে 1টা $\binom{10}{1} = 10$

→ 5 যায়গার ভিতর 3 জায়গা $\binom{5}{3} = 10$

→ বাকি 2 জায়গায় 2টা $= 9 \times 8 = 72$

So, $10 \times 10 \times 72 = 7200$

$E_e = 720$

Sub.: _____ SAT SUN MON TUE WED THU FRI

DATE: / /

⑤ Full House (aaabb):

→ 10টির মধ্যে থেকে 1 টি (3টার জন্য) and বাকি প্রথম থেকে (জোড়ার জন্য) 9টি $\Rightarrow 10 \times 9 = 90$

→ 5 টির জায়গায় 3টি নাও $\binom{5}{3} = 10$

→ বাকি 2 জায়গায় জোড়াটি বসাবে $\binom{2}{2} = 1$

So, $90 \times 10 \times 1 = 900$

$E_6 = 90$

⑥ Four of a kind (aaaab):

$$\binom{10}{1}\binom{5}{4}\binom{9}{1} = 450$$

$E_6 = 45$

Sub.: _____ SAT SUN MON TUE WED THU FRI

DATE: / /

⑦ Five of kind (aaaaa):

$$\binom{10}{1} = 10$$

$E_e = 1$

(F)

Combination Distribution i	Observed Distribution O_i	Expected E_i	$\frac{(O_i - E_i)^2}{E_i}$
Five of Different	3075	3024	0.8601
1 Pairs	4935	5040	2.1875
2 Pairs	1135	1080	2.8009
3 of a kinds	695	720	0.8680
Full houses	105	90	2.5
Four of a kind	54	45	1.8
5 of a kind	01	01	0
	10,000	10,000	10.0907

Sub.: _____

SAT SUN MON TUE WED THU FRI

DATE: / /

Degree of Freedom (df) = 7 − 1 = 6

Critical value for $\alpha = 0.01$ and $df = 6$ is 16.812

So, $\chi^2 (10.0907) < \chi^2_{critical} (16.812)$. The random value of 10,000 is independent.

Sub.: _____

SAT SUN MON TUE WED THU FRI

DATE: / /

$$R_{n+1} = (aR_n + c) \bmod m$$

- R_{n+1} → next random number
- a → multiplier
- R_n → current random number
- c → increment
- m → modulus

Example 4.1:

Mixed multiplicative congruential (MMC) Generating

$a=13$, $c=1$ and $m=19$. Let $r_0=1$

$r_1 = (1\times13+1) \bmod 19 = 14 \bmod 19 =$ residue 14

$r_2 = (14\times13+1) \bmod 19 = 183 \bmod 19 =$ " 12

$r_3 = (12\times13+1) \bmod 19 = 157 \bmod 19 =$ " 5

$r_4 = (5\times13+1) \bmod 19 = 66 \bmod 19 =$ " 9

$r_5 = (9\times13+1) \bmod 19 = 118 \bmod 19 =$ " 4

$r_6 = (4\times13+1) \bmod 19 = 53 \bmod 19 =$ " 15

Sub.: _____

SAT SUN MON TUE WED THU FRI

DATE : / /

This sequence of random number between 0 and 18 both inclusive will continue repeating. The number 18, which is one less than modulo is called Euler function.

Random number between 0 and 1 can be generated by

$$R_i = \frac{r_i}{m}; \quad i = 1, 2, 3, \ldots$$

which gives the sequence as-

$$R_1 = \frac{14}{19} = 0.7368$$

$$R_2 = \frac{12}{19} = 0.6316$$

$$R_3 = \frac{5}{19} = 0.6315$$

$$R_4 = \frac{9}{19} = 0.4737$$

$$R_5 = \frac{4}{19} = 0.2105$$

⋮

etc

Sub.: _____ SAT SUN MON TUE WED THU FRI

DATE: / /

(6) Multiplicative Congruential (MC) Generator:

$$r_{i+1} = a r_i \bmod m$$

Again taking $a = 13$; $m = 19$ and seed $r_0 = 1$

$r_1 = 13 \bmod 19 = 0$; residue $13 = 13$

$r_2 = (13 \times 13) \bmod 19 = 8$; " $18_1 = 18$

$r_3 = (8 \times 13) \bmod 19 = 12$; " $6 = 6$

$r_4 = (13 \times 6) \bmod 19 = 4$; " $2 = 2$

$r_5 = (2 \times 13) \bmod 19 = 1$; " $7 = 7$

$r_6 = (7 \times 13) \bmod 19 = 4$; " $15 = 15$

The sequence of numbers obtained is 1, 13, 18, 6, 2, 7, 15 ....

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

⑥ Additive Congruential Generator:

$$r_{i+1} = (r_i + 5) \bmod m$$

$m = 19$; $b = 11$; $r_0 = 1$

$r_1 = (1 + 11) \bmod 19 = 12$

$r_2 = (12 + 11) \bmod 19 = 4$

$r_3 = (4 + 11) \bmod 19 = 15$

$r_4 = (15 + 11) \bmod 19 = 7$

$r_5 = (7 + 11) \bmod 19 = 18$

$r_6 = (18 + 11) \bmod 19 = 10$

....

Example 4.2

Null Hypothesis (H_0): Numbers গুলো Uniformly distributed

Level of significance (α): 0.05 (৯৫%. নিশ্চিত হতে চাই)

i	1	2	3	4	5	6	7	8	9	10
R_i	0.11	.23	.24	.41	.50	.61	.64	.65	.86	.89
i/N	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$i/N - R_i$	—	—	.06	—	.0	—	.06	.15	.04	.11
$R_i - (i-1)/N$	.11	.13	.04	.11	.10	.11	.04	—	.06	—

Now, $D = \max(D^+, D^-) = \max(.15, .13) = .15$

$N = 10$ and $\alpha = 0.05$ Table এর value 0.410

Condition: If calculated D < critical D then it is Null Hypothesis.

As, $0.15 < 0.410$, we can say that the numbers are Uniform Distributed.

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Example - 4.3

Class	Count	Frequency	Diff	$(Diff)^2$
$0 < n \leq 10$	𝍸 𝍸 III	13	3	9
$10 < n \leq 20$	𝍸 III	8	2	4
$20 < n \leq 30$	𝍸 I III	9	1	1
$30 < n \leq 40$	𝍸 𝍸 III	13	3	9
$40 < n \leq 50$	𝍸 II	7	3	9
$50 < n \leq 60$	𝍸 𝍸 III	13	3	9
$60 < n \leq 70$	𝍸	5	5	25
$70 < n \leq 80$	𝍸 𝍸 II	12	2	4
$80 < n \leq 90$	𝍸 𝍸 II	12	2	4
$90 < n \leq 100$	𝍸 III	8	2	4
				78

$$\text{Chi square} = \frac{78}{10} = 7.8$$

Degree of Freedom $(df) = n - 1 = 9$

Confidence Level $= 95\%$ $(\alpha = 0.05)$

From the table, $df = 9$ and $\alpha = 0.05$ then the critical value is $= 16.9$.

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

As $7.8 < 16.9$ than we can say that the set of random numbers is acceptable and so far it is uniformity in distribution is concerned.

(4.13) Autocorrelation (সংখ্যা গুলোর মধ্যে কোনো pattern আছে কিনা).

এখানে দেখা যাচ্ছে যে, ১টি number তার পরের number এর উপর depend করে কিনা তার উপর নির্ভর করে. (0.49, 0.95) ; (0.95, 0.62) pair এভাবে হবে.

Total class:

① ≤ 0.33 (small)

② ≤ 0.67 (medium)

③ ≤ 1.0 (large)

As the pair is (R_1, R_2) than there are $3 \times 3 = 9$ cell. (small, small); (small, Medium); (medium, large)

Total pair $= 73 - 1 = 72$ | Expected frequency $(E_i) = 72 \div 9 = 8$

Sub.: _____ SAT SUN MON TUE WED THU FRI

DATE: / /

class	Count	Frequency	Diff	$(Diff)^2$
$R_1 \leq 0.33$ & $R_2 \leq 0.33$	𝍸 IIII	9	1	1
$R_1 \leq 0.67$ & $R_2 \leq 0.33$	𝍸 III	7	1	1
$R_1 \leq 1$ & $R_2 \leq 0.33$	𝍸 I	6	2	4
$R_1 \leq 0.33$ & $R_2 \leq 0.67$	𝍸 I	6	2	4
$R_1 \leq 0.67$ & $R_2 \leq 0.67$	𝍸 III	8	0	0
$R_1 \leq 1$ & $R_2 \leq 0.67$	𝍸 IIII	9	1	1
$R_1 \leq .33$ & $R_2 \leq 1$	𝍸 III	7	1	1
$R_1 \leq .67$ & $R_2 \leq 1$	𝍸 IIII	9	1	1
$R_1 \leq 1$ & $R_2 \leq 1$	𝍸 𝍸 I	11	3	9
		72		24

$$\text{Chi-square} = \frac{24}{8} = 3$$

Degree of Fredom (df): $9-2=7$

Critical value : 95% so, $\alpha = 0.05$. so, value = 14.067

As $3 < 14.067$ so, there is no serial Accommodation. The numbers are independent and random.

Sub.: 19-20

4(b) (Pursuit)

SAT SUN MON TUE WED THU FRI

DATE: / /

Given that, $S = 30$ km/min

$$(x_f, y_f) = (50, 50)$$

If $Dist(t) \le 10$ KM then the pursuit ends otherwise abandoned.

$$Dist(t) = \sqrt{(y_b - y_f)^2 + (x_b - x_f)^2}$$

$$\sin\theta = \frac{y_b - y_f}{D(t)} \; ; \; \cos\theta = \frac{x_b - x_f}{D(t)}$$

$$x_f(t+1) = x_f(t) + S\cos\theta \; ; \; y_f(t+1) = y_f(t) + S\sin\theta$$

$A_5 = \sqrt{(A_1 - A_2)^2 + (A_3 - A_4)^2}$

$A_6 = \frac{A_1 - A_3}{A_5}$

$A_7 = \frac{A_2 - A_4}{A_5}$

$A_8 = A_3 + 30 \times A_6$

$A_9 = A_4 + 30 \times A_7$

	A1	A2	A3	A4	B1	B2	B3	B4	B5
t	$x_b(t)$	$y_b(t)$	$x_f(t)$	$y_f(t)$	$D(t)$	$\cos\theta$	$\sin\theta$	$x_f(t+1)$	$y_f(t+1)$
0	160	0	50	50	120.83	0.91	-0.413	77.3	37.61
1	161	4	77.3	37.61	90.196	0.9279	-0.372	105.13	26.431
2	167	9	105.13	26.431	64.278	0.9625	-0.271	134	18.295
3	172	11	134	18.295	38.693	0.982	-0.188	163.46	12.639
4	181	14	163.46	12.639	17.592	0.997	0.0773	193.37	14.959
5	189	24	193.37	14.959	10.04	-0.435	0.90	180.31	41.969

Sub.: _____

SAT SUN MON TUE WED THU FRI

DATE: / /

6	199	34	180.31	41.969	20.317	0.9198	-0.392	207.9	30.202
7	215	41	207.9	30.202	12.923	0.5494	0.6355	224.38	55.268
8	239	52	224.36	55.268	14.98	0.9759	-0.218	253.65	48.723
9	245	45	253.65	48.723	9.41	-0.918	-0.395	226.09	36.862

After 9 min the pursuit end at distance

~~8.56 km.~~ 9.41 km.

12-20

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

5(c)

Given that:

$k_1 = 0.025$; $k_2 = 0.01$; $t = 0$;

$C_1 = 50$; $C_2 = 25$; $C_3 = 0$; $\Delta t = 0.2$ min

$$C_1(t+\Delta t) = C_1(t) + [k_2 C_3(0) - k_1 C_1 C_2]\Delta t$$

$$C_2(t+\Delta t) = C_2(t) + [k_2 C_3 - k_1 C_1 C_2]\Delta t$$

$$C_3(t+\Delta t) = C_3(0) + [2k_1 C_1(0) C_2(0) - 2k_2 C_3(0)]\Delta t$$

Calculation:

A1 ; A2, A3

↓ ↓ ↓

C_1 C_2 C_3

$C_1 \rightarrow B1 = A1 + (((0.01 \times A3) - (0.025 A1 A2)) \times 0.2)$

$C_2 \rightarrow B2 = A2 + (((0.01 \times A3) - (0.025 \times A1 \times A2)) 0.2)$

$C_3 \rightarrow B3 = A3 + (((2 \times 0.025 A1.A2) - (2 \times 0.01 \times A3)) \times 0.2)$

Sub.: _____ SAT SUN MON TUE WED THU FRI

DATE: / /

T	C_1	C_2	C_3	T	C_1	C_2	C_3
0	50	25	0	1.8	29.29	4.303	41.358
0.2	43.75	18.75	12.5	2	28.742	3.75	42.45
0.4	39.673	14.673	20.653	2.2	28.28	3.29	43.36
0.6	36.80	11.8	26.39	2.4	27.901	2.9115	44.116
0.8	34.681	9.681	30.626	2.6	27.583	2.593	44.75
1.0	33.063	8.063	33.86	2.8	27.314	2.324	45.286
1.2	31.797	6.7977	36.39	3	27.087	2.097	45.739
1.4	30.78	5.78	38.405				
1.6	29.96	4.9672	40.03				

50, $C_1 = 27.087$

$C_2 = 2.097$

$C_3 = 45.739$