Chapter -2

Example-2.1

Area of an irregular figure.

F= area of irregular figure

A= area of square to be consider

M= Points within shape

N= Total random points

So, here $\frac{F}{A} = \frac{M}{N}$

$\Rightarrow F = \frac{M}{N} \times A$

Random number $\longrightarrow$ (0-1)

we will consider $9 \times 9\ cm^2$ regular shape

So, $A = 9 \times 9 = 81\ cm^2$

Sub.:

DATE: / /

For figure-1:

$N = 50$ and $M = 17$

So, $\frac{M}{N} = \frac{17}{50} = 0.34$

But the real ratio is $= 0.3126$

For fig-2:

$N = 84$ and $M = 26$

So, $\frac{M}{N} = \frac{26}{84} = 0.3095$

Which is near to the 0.3126.

So, we can say that, when we increase the point the accuracy is increase.

So, area of $F = \frac{M}{N} \times A$

$= 0.3095 \times 81 = 25.07 \text{ cm}^2$

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

For random number

$$x = (\text{random number}) \times 9$$

$$y = (\quad " \quad) \times 9$$

$x_1 = 0.96 \times 9 = 8.64$ $y_1 = 0.42 \times 9 = 3.78$

$x_2 = 0.71 \times 9 = 6.39$ $y_2 = 0.10 \times 9 = 0.9$

$x_3 = 0.03 \times 9 = 0.27$ $y_3 = 0.03 \times 9 = 2.7$

Example-2.2

Gambling Game:

① An unbised coin is repeatedly flipped

② For each flip you have to pay RS.1

③ when difference between heads and tails tossed become 3, you get RS. 8.

④ Thus if the required difference is obtained in less than 8 flips, you win some money and if it is in more than 8 flip you lose.

→ 50% possibility (head or tail)

→ Random number (0-9)

(0-4) for Head(H) and

(5-9) for Tail(T)

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Game No	Serial No	Random Number	Head or Tail	Cumulative Heads	Cumulative Tails	Difference
1	1	5	T	0	1	1
	2	9	T	0	2	2
	3	3	H	1	2	1
	4	6	T	1	3	2
win RS 1	5	4	H	2	3	1
	6	8	T	2	4	2
	7	6	T	2	5	3
2	1	8	T	0	1	1
	2	1	H	1	1	0
	3	5	T	1	2	1
Lose RS-1	4	2	H	2	2	0
	5	4	H	3	2	1
	6	0	H	4	2	2
	7	6	T	4	3	1
	8	3	H	5	3	2
	9	1	H	6	3	3
3	1	2	H	1	0	1
	2	9	T	1	1	0
win RS 3	3	4	H	2	1	1
	4	2	H	3	1	2
	5	3	H	4	1	3

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

Total money paid = 7+9+5 = 21

Returned money = 21 − loss + win

= 21 − 1 + 4

= 24

or

= total game × 8

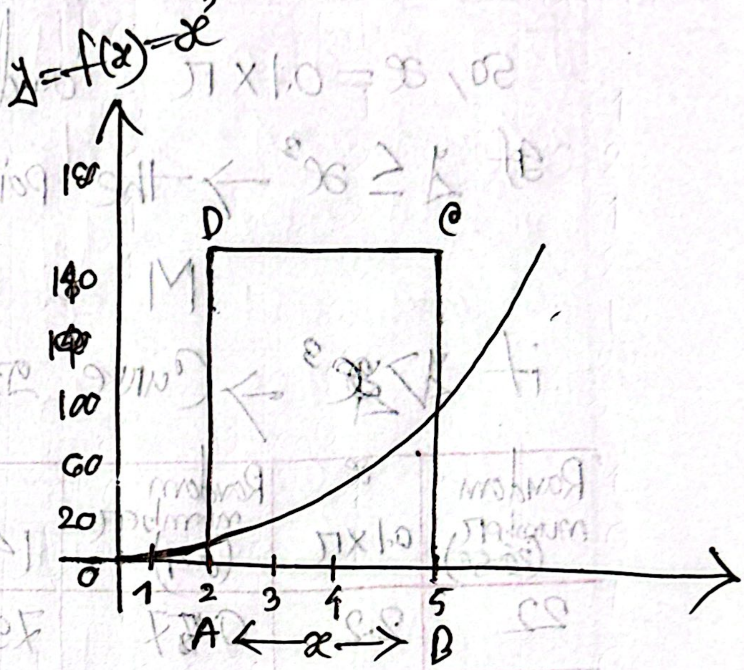
= 3×8

= 24

Each game on the average required $\frac{21}{3}$ = 7 Aits

Example – 2.3

Numerical Integration:



$$I = \int_2^5 x^3 \, dx$$

$$= \int_a^b f(x) \, dx$$

here, $f(x) = x^3$

So, $I = \int_2^5 x^3 \, dx$

$$= \left[\frac{x^4}{4} \right]_2^5$$

$$= \frac{625 - 16}{4}$$

$$= 152.25$$

Here the limit between (2-5). The ratio of the shaded area to the area A of rectangle ABCD is M to N

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

The Area is, $A = 140 \times 3 = 420$

so, $x = 0.1 \times r$ and $y = 140 \times r_n$

If $y \le x^3 \rightarrow$ the point is under the curve and M is increased

if $y > x^3 \rightarrow$ Curve এর উপরে

Random number (20-50)	x $0.1 \times r$	Random number (0-1)	y $140 \times r_n$	x^3	M	N
22	2.2	0.57	79.8	10.64	0	1
25	2.5	0.18	25.2	15.625	0	2
18	1.8	0.00	0	5.832	1	3
45	4.5	0.90	126	91.125	1	4
25	2.5	0.05	7	15.625	2	5
27	2.7	0.77	107.8	19.683	2	6
48	4.8	0.66	92.4	110.592	3	7
43	4.3	0.10	14	79.507	4	8
40	4	0.76	106.4	64	4	9
47	4.7	0.42	58.8	103.823	5	10

so, $I = \frac{5}{10} \times 140 \times 3 = 210$

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Example 2.4

Area of the quadrant $= \frac{\pi r^2}{4} = \frac{\pi (1)^2}{4} = \frac{\pi}{4}$

The equation is, $x^2+y^2 \leq 1$; $x, y \geq 0$

So, M = accepted points

N = total points

So, $\frac{M}{N} = \frac{\pi}{4}$

and $\pi = 4 \times \frac{M}{N}$

Random numbers, R_1 and $R_2 \rightarrow (0-1)$.

$R_1^2 + R_2^2 \leq 1$

$\therefore R_2 \leq \sqrt{1-R_1^2}$ (In হবে)

R_1	R_2	$\sqrt{1-R_1^2}$	In/out	R_1	R_2	$\sqrt{1-R_1^2}$	In/out
0.82	0.95	0.572	out				
0.34	0.14	0.94	In				
0.20	0.84	0.97	In				
0.58	0.14	0.81	In				
0.52	0.03	0.85	In				
0.51	0.60	0.86	In				
0.34	0.38	0.94	In				
0.72	0.11	0.69	In				
0.50	0.55	0.86	In				
0.08	0.21	0.99	In				

out of 10 points
9 lies with in
quadrant, so

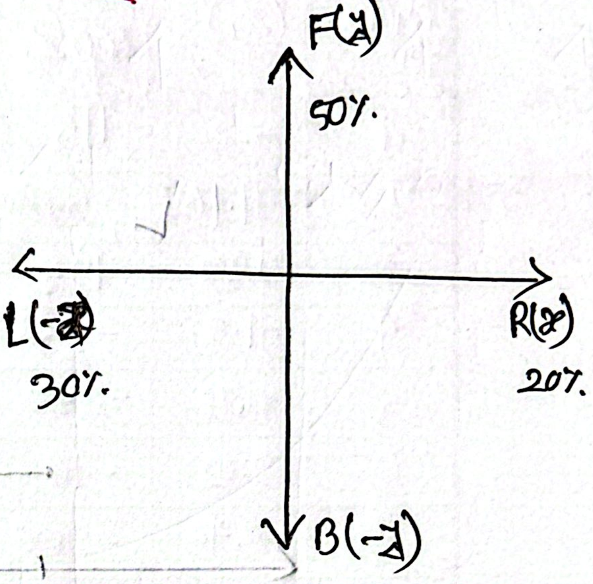
$$\pi = \frac{9}{10} \times 4$$

$$= 3.6$$

Example 2.5

Random walk

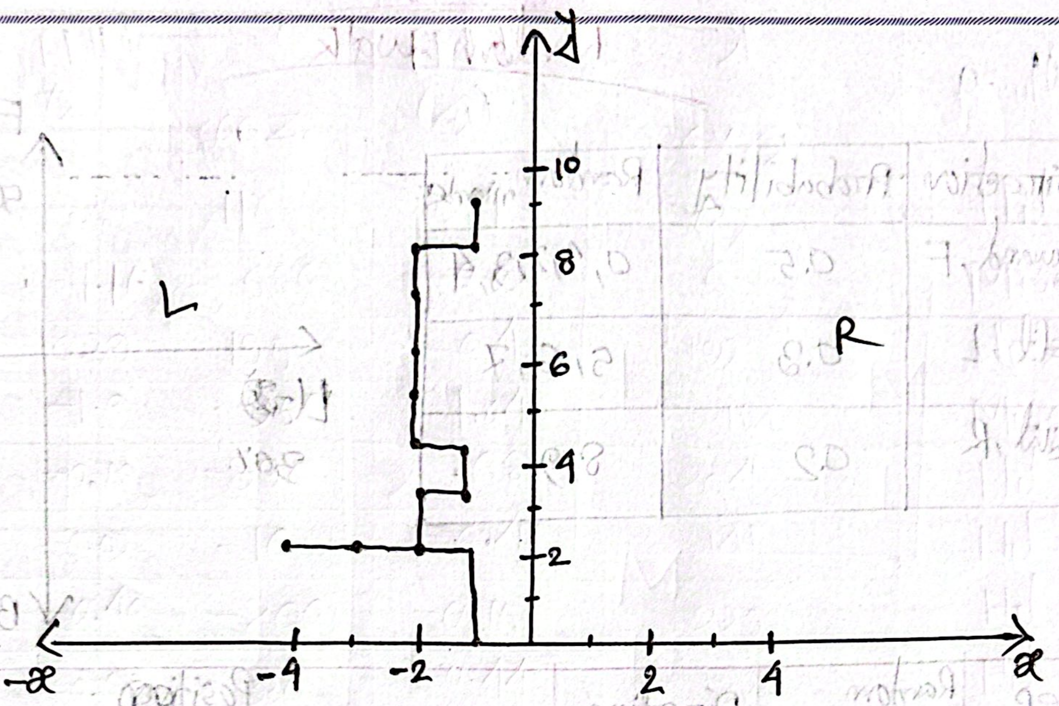
Direction	Probability	Random number
Forward, F	0.5	0,1,2,3,4
Left, L	0.3	5,6,7
Right, R	0.2	8,9



Step	Random Number	Direction	Position x	Position y
1	6	L	-1	0
2	2	F	-1	1
3	0	F	-1	2
4	6	L	-2	2
5	8	R	-1	2
6	5	L	-2	2
7	7	L	-3	2
8	7	L	-4	2
9	9	R	-3	2
10	8	R	-2	2
11	4	F	-2	3
12	8	R	-1	3
13	2	F	-1	4
14	6	L	-2	4
15	2	F	-2	5
16	1	F	-2	6
17	3	F	-2	7
18	0	F	-2	8
19	8	R	-1	8
20	4	F	-1	9

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /



Example-27

A normal random variable x is given by

$$x = \mu + \sigma z$$

here,

μ = true mean of distribution

σ = standered daviation of x

z = normal random number

$$\left.\begin{aligned} x &= \mu_x + \sigma_x z_x \\ y &= \mu_y + \sigma_y z_y \end{aligned}\right\} \text{Coordinates}$$

Center points, $\mu_x = \mu_y = 0$

The given value of standered deviation in x and y directions are 500 m and 300 m respectively

$x_1 = 500 Z_x$

$y_1 = 300 Z_y$

500 ও 300 — ভিতরে থাকলেই hit

Bomb strike	RNN (z_x)	x $500z_x$	RNN (z_y)	y $300 \times z_y$	Result
1	.23	115	.24	72	Hit
2	-1.16	-580	-0.02	-6	Miss
3	0.39	195	-6.4	192	Hit
4	-1.9	-950	-1.04	-312	Miss
5	-0.78	-390	.68	204	Hit
6	-0.02	-10	-0.47	-141	Hit
7	-0.40	-200	-0.75	-225	Hit
8	-0.66	-330	-0.44	-132	Hit
9	1.41	705	1.21	363	Miss
10	0.07	35	-0.08	-24	Hit
11	0.53	265	-1.56	-468	Miss
12	0.03	15	1.49	447	M
13	-1.19	-595	-1.19	-357	M
14	0.11	55	-1.86	-558	M
15	0.16	80	1.21	363	M
16	-0.82	-410	0.75	225	H
17	0.42	210	-1.50	-450	M
18	-0.89	-445	0.19	57	H
19	0.49	245	-1.44	-432	M
20	-0.21	-105	0.65	195	Hit

SAT SUN MON TUE WED THU FRI

Sub.: DATE: / /

Mid Question

Direction	Probability	Distance	Random number range	
Backward	15%	50cm	0-14	$y = y - 50$
Forward	40%	90cm	15-54	$y = y + 90$
Right	20%	60cm	55-74	$x = x + 60$
Left	25%	40cm	75-99	$x = x - 40$

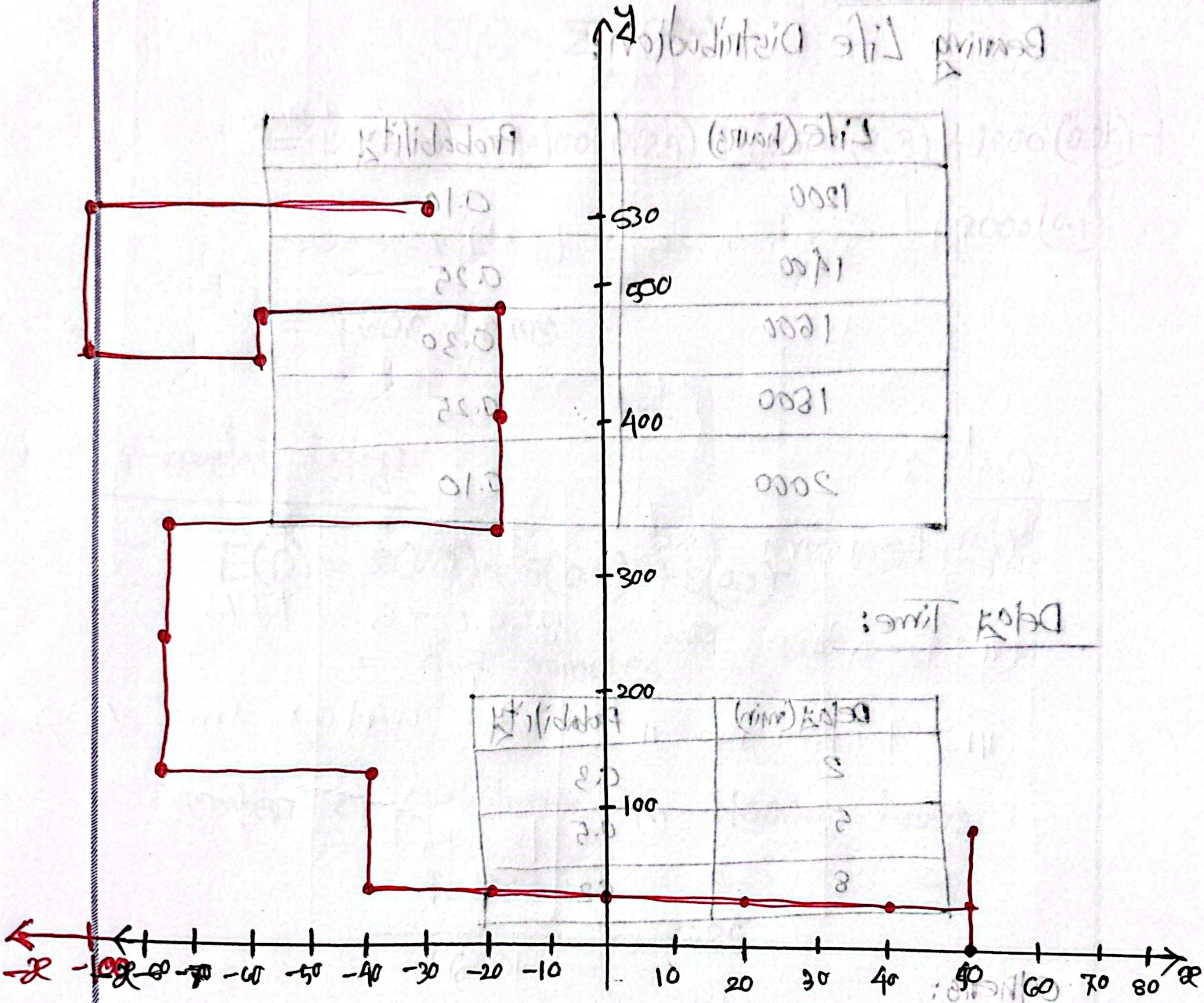
Step	RNN	Direction	(x,y)	Step	RNN	Direction	(x,y)
1	67	R	(60,0)	11	45	F	-80,220
2	23	F	(60,90)	12	29	F	-80,310
3	01	B	(60,40)	13	68	R	-20,310
4	95	L	20,40	14	21	F	-20,400
5	84	L	-20,40	15	15	F	-20,490
6	56	R	40,40	16	86	L	-60,490
7	77	L	0,40	17	00	B	-60,440
8	76	L	-40,40	18	89	L	-100,440
9	18	F	-40,130	19	45	F	-100,530
10	82	L	-80,130	20	67	R	-40,530

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

Fig:



Sub.: SAT SUN MON TUE WED THU FRI DATE: / /

Bearing

Given data—

Bearing Life Distribution:

Life (hours)	Probability
1200	0.10
1400	0.25
1600	0.30
1800	0.25
2000	0.10

Delay Time:

Delay (min)	Probability
2	0.3
5	0.5
8	0.2

Others:

Change 1 bearing = 15 min
" 2 " = 25 "
" 3 " = 35 "

Downtime cost = $5 per minute
Technician " = $30 per hour
cost of one bearing = $30
Simulation period = 10,000 hours

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Total cost = 540 + 1191 + 105

= 1836

So, savings = 2448 − 1836 = 612

saved by proposed policy.

Bearing Life	Probabilities	Cumulative Probability	RNN
1200	0.10	0.10	01−10
1400	0.25	0.35	11−35
1600	0.30	0.65	36−65
1800	0.25	0.90	66−90
2000	0.10	1.00	91−100

Delay time	Probability	cumulative	RNN
2	0.3	0.3	1−3
5	0.5	0.8	4−8
8	0.2	1.0	9−0

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Bearing 1					2					3				
RD	Life	clock	RD	Delay	RD	Life	clock	RD	Delay					
25	1400	1400	9	8	65	1600	1600	5	5	94	2000	2000	7	5
82	1800	3200	8	5	91	2000	3600	0	8	87	1800	3800	0	8
16	1400	4600	0	8	30	1400	5000	5	5	63	1600	5400	6	5
14	1400	6000	5	5	66	1800	6800	6	5	66	1800	7200	2	2
82	1800	7800	2	2	32	1400	8200	5	5	30	1400	8600	2	2
45	1600	9400	3	2	29	1400	9600	3	2	09	1800	10400	7	5
09	1200	10600	7	5	08	1200	10800	2	2					
Total Delay = 35					Total delay 32					Total delay 27				

20 bearing have been replaced

cost of " $= 20 \times 30 = 600$

cost of downtime $= \left((35+32+27)+(20\times15)\right)\times 5$

$= 1970$

cost " technician $= (20\times15)\times\frac{30}{60} = 150$

Total $= 2720$

Sub.:

SAT SUN MON TUE WED THU FRI

DATE: / /

Bearing 1 Life	Bearing 2 Life	3 Life	First failure	Cumulative Life	RD	Delay
1400	1800	2000	1400	1400	4	5
1800	2000	1800	1800	3200	6	5
1400	1400	1600	1400	4600	7	5
1400	1800	1800	1400	6000	3	2
1800	1400	1400	1400	7400	4	5
1600	1400	1800	1400	8800	6	5
1200	1200	1600	1200	10000	7	5

Total delay = 32

cost of bearings = $21 \times 30 = 630$

" " downtime = $(32 + (7 \times 35)) \times 5$

= 1385

" " Technician = $7 \times 35 \times \frac{30}{60}$

= 122.5

Total = 2137.5

Exercise 2(b)

$$I = \int_1^5 \frac{x^4}{3}\,dx$$

$$= \left[\frac{x^5}{15}\right]_1^5 = 208.3$$

$y \leq x^4/3$

Area of rectangle $= 208.3 \times 4$

$= 833.2$

Random	$x=0.1\times r$	Random	$y=208.3\times r$	$\frac{x^4}{3}$	N'	N
48	4.8	.52	108.3	176.94	1	1
36	3.6	.18	37.5	55.98	2	2
44	4.4	.30	62.49	124.93	3	3
41	4.1	.88	183.3	94.192	3	4
46	4.6	.21	43.74	149.248	4	5
18	1.8	.25	52.07	3.499	4	6
32	3.2	.57	118.73	34.95	4	7
41	4.1	.82	170.80	94.192	4	8
23	2.3	.9	187.4	9.32	4	9
23	2.3	.63	131.229	9.23	4	10

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

So, $I = \frac{4}{10} \times 833.2 = 333.28$

Problem – 6

$x_1 = 500 Z_x$

$y_1 = 300 Z_y$

Strike	RNN	$x_1 = 500 Z_x$	RNN	$y_1 = 300 Z_y$	Result
1	0.23	115	0.24	72	H
2	−1.16	−580	−0.02	−6	Miss
3	0.39	195	0.64	192	H
4	−1.90	−950	−1.04	−312	M
5	−0.78	−390	0.68	104	H
6	−0.02	−10	−0.47	−141	H
7	−0.40	−200	−0.75	−225	H
8	−0.66	−330	−0.44	−132	H
9	1.41	705	1.21	363	M
10	0.07	35	−0.08	−24	H

Sub.: SAT SUN MON TUE WED THU FRI

DATE: / /

Exercise-7

Given that,

$\mu_x = 50$, $\sigma_x^2 = 65$

$\mu_y = 100$, $\sigma_y^2 = 100$

$\mu_z = 150$ $\sigma_z^2 = 150$

So, $x = 50 + 8.062 Z_1$

$y = 100 + 10 Z_2$

$z = 150 + 12.24 Z_3$

RNN	x	RNN	y	RNN	z	$x+y$	$w = \frac{x+y}{z}$
-0.46	46.29	0.09	100.9	0.59	157.22	147.19	0.936
-1.05	41.53	0.56	105.6	-0.67	141.799	147.13	1.0375
-0.15	48.79	-0.02	99.8	0.41	155.01	148.59	
-0.81	43.46	3.10	131	0.51	156.24	174.46	
-0.74	44.03	-0.44	95.6	1.53	168.72	139.63	