Subject : D.S. Hira Book

Date :

Time :

Chapter - 02

Examples

(2.1) Area of an irregular figure

F = Area of irregular figure

A = Area of square to be considered

M = Points within shape

N = Total random points

Here, $\frac{F}{A} = \frac{M}{N} \Rightarrow F = \frac{M}{N} \times A$

Random number → (0 - 1)

We will consider 9x9 cm^2 regular shape

From Random number table (দেওয়া থাকবে)

$x_1 = 0.96 \times 9 = 8.64$ $\quad$ $y_1 = 0.42 \times 9 = 3.78$

$x_2 = 0.71 \times 9 = 6.39$ $\quad$ $y_2 = 0.10 \times 9 = 0.9$

$x_3 = 0.03 \times 9 = 0.27$ $\quad$ $y_3 = 0.30 \times 9 = 2.7$

Area, $F = \frac{26}{84} \times 9 \times 9 = 25.07 \, cm^2$ (Basic) (main area)

$A = 9 \times 9 = 81 \, cm^2$

$$\text{Ratio} = \frac{25.07}{81} \times 100$$

$$= 30.95\%$$

Subject :

Date :
Time :

Case 1: $A = \frac{17}{50} \times 81 = 27.54 \text{ cm}^2$

Difference $= 27.54 - 25.07 = 2.47 \text{ cm}^2$

Case 2: $A = \frac{45}{140} \times 81 = 26.04 \text{ cm}^2$

Difference $= 26.04 - 25.07 = 0.97 \text{ cm}^2$

∴ Accuracy grows higher with increasing points.

(2.2) Gambling Game

* An unbiased coin is repeatedly flipped
* For each flip you have to pay Rs. 1.
* When difference between heads tossed and tails tossed become 3, you get Rs. 8.
* Thus if the required difference is obtained in less than 8 flips, you win some money and if it is in more than 8 flips you lose.

→ 50% possibility (head or tail)

→ Random number (0–9)

(0–4) for H, (5–9) for T

Subject :

Date :

Time :

Game No.	Serial No.	Random Number	Head or Tail	Cumulative Heads	Cumulative Tails	Difference
1	1	5	T	0	1	1
Win Rs. 1	2	9	T	0	2	2
	3	3	H	1	2	1
	4	6	T	1	3	2
	5	4	H	2	3	1
	6	8	T	2	4	2
	7	6	T	2	5	3
2	1	8	T	0	1	1
Lose Rs. 1	2	1	H	1	1	0
	3	5	T	1	2	1
	4	2	H	2	2	0
	5	4	H	3	2	1
	6	0	H	4	2	2
	7	6	T	4	3	1
	8	3	H	5	3	2
	9	1	H	6	3	3
3	1	2	H	1	0	1
Win Rs. 3	2	9	T	1	1	0
	3	4	H	2	1	1
	4	2	H	3	1	2
	5	3	H	4	1	3
4	1	3	H	1	0	1
Lose Rs. 3	2	3	H	2	0	2
	3	7	T	2	1	1
	4	1	H	3	1	2
	5	8	T	3	2	1
	6	0	H	4	2	2
	7	6	T	4	3	1
	8	2	H	5	3	2
	9	9	T	5	4	1
	10	3	H	6	4	2
	11	4	H	7	4	3

(প্রতিটির game এ total 8 flips count হবে।

Game 1 এ 7 টি flips এ difference 3

তাহলে হবে (8-7) ⇒ Rs. 1 win করছি।

Game 4 এ 11 টি flips এ difference 3 |

তাহলে হবে (11-8) ⇒ Rs. 3 lose করছি।)

∴ Total money paid = 7 + 9 + 5 + 11

= 32

Returned money = 32 − lose + win

= 32 − 4 + 4

= 32 এভাবে করলে হবে

নয়তো

= total game no. × 8

= 4 × 8

= 32

per game এ total flip

Each game on the average requires $\frac{32}{4} = 8$ flips.

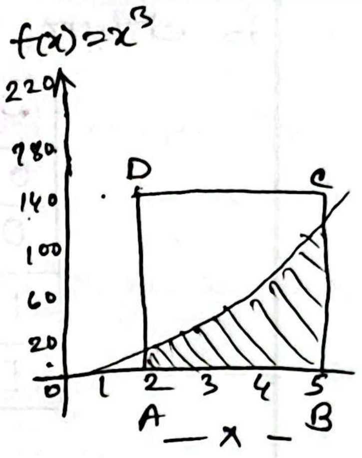
Subject :

Date :

Time :

(2.3) Numerical Integration

$$I = \int_2^5 x^3\,dx = \int_a^b f(x)\,dx$$



$$f(x) = x^3$$

$$I = \int_2^5 x^3\,dx = \left[\frac{x^4}{4}\right]_2^5 = 150$$

Here the limit between (2–5). The ratio of the shaded area to the area A of rectangle ABCD is M to N.

$$I = \frac{M}{N} \times A \qquad A = 140 \times 3 = 420 \rightarrow x^{[5-2]}$$

$y \le x^3$ – (২ বার বাড়বে)

Random Number (20–50)	x $0.1 \times R_n$	Random number (0–1)	y $140 \times R_n$	x^3	M	N serial
22	2.2	.57	79.8	10.648	0	1
25	2.5	.18	25.2	15.625	0	2
18	1.8	.00	0	5.832	1	3
45	4.5	.90	126	91.125	1	4
25	2.5	.05	7	15.625	2	5
27	2.7	.77	107.8	19.683	2	6
48	4.8	.66	92.4	110.592	3	7
43	4.3	.10	14	79.507	4	8
40	4.0	.76	106.4	64	4	9
47	4.7	.42	58.8	103.823	5	10

$$I = \frac{5}{10} \times 140 \times 3$$

$$= 210$$ (আরও বেশি simulate করলে value কমে আসতো। তবে ১৫০ই exact value)

ship

Subject :

Date :

Time :

2.4 Pi value

Area of circle $= \frac{\pi D^2}{4} = \frac{\pi (2r)^2}{4} = \pi r^2$

Equation, $x^2 + y^2 \le 1$; $x, y \ge 0$

Random numbers, R_1 and $R_2 \rightarrow (0-1)$

$R_1^2 + R_2^2 \le 1$

$\therefore R_2 \le \sqrt{1 - R_1^2}$ (In 275)

$\frac{M}{N}$ will take approch and get value $\frac{\pi}{4}$.

$M \rightarrow$ lie within the quadrant

$N \rightarrow$ total random numbers

R_1	R_2	$\sqrt{1-R_1^2}$	In/out	R_1	R_2	$\sqrt{1-R_1^2}$	In/out
.82	.95	0.5724	Out	.76	.18	0.6499	In
.34	.14	0.9404	In	.33	.81	0.9439	In
.20	.84	0.9798	In	.35	.96	0.9367	Out
.58	.14	0.8146	In	.29	.57	0.9570	In
.52	.03	0.8542	In	.95	.42	0.3122	Out
.51	.60	0.8602	In	.84	.51	0.5426	In
.34	.38	0.9404	In	.34	.68	0.9404	In
.72	.11	0.6939	In	.85	.11	0.5268	In
.50	.55	0.8660	In	.58	.92	0.8146	Out
.08	.21	0.9968	In	.69	.59	0.7238	In

Out of 20 points, 16 lie within the quadrant

$\therefore \frac{\pi}{4} = \frac{16}{20}$

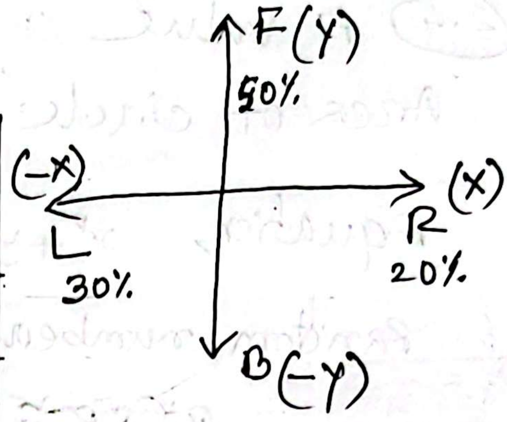
$\Rightarrow \pi = \frac{4}{5} \times 4$

$= 3.2$

Subject : Date : Time :

(2.5) Random walk

Direction	Probability	Random Numbers
Forward, F	0.5	0, 1, 2, 3, 4
Left, L	0.3	5, 6, 7
Right, R	0.2	8, 9



Step	Random Number	Direction	Position	
			x	y
1	6	L	-1	0
2	2	F	-1	1
3	0	F	-1	2
4	6	L	-2	2
5	8	R	-1	2
6	5	L	-2	2
7	7	L	-3	2
8	7	L	-4	2
9	9	R	~~-4~~ -3	2
10	8	R	-2	2
11	4	F	-2	3
12	8	R	-1	3
13	2	F	-1	4
14	6	L	-2	4
15	2	F	-2	5
16	1	F	-2	6
17	3	F	-2	7
18	0	F	-2	8
19	8	R	-1	8
20	4	F	-1	9

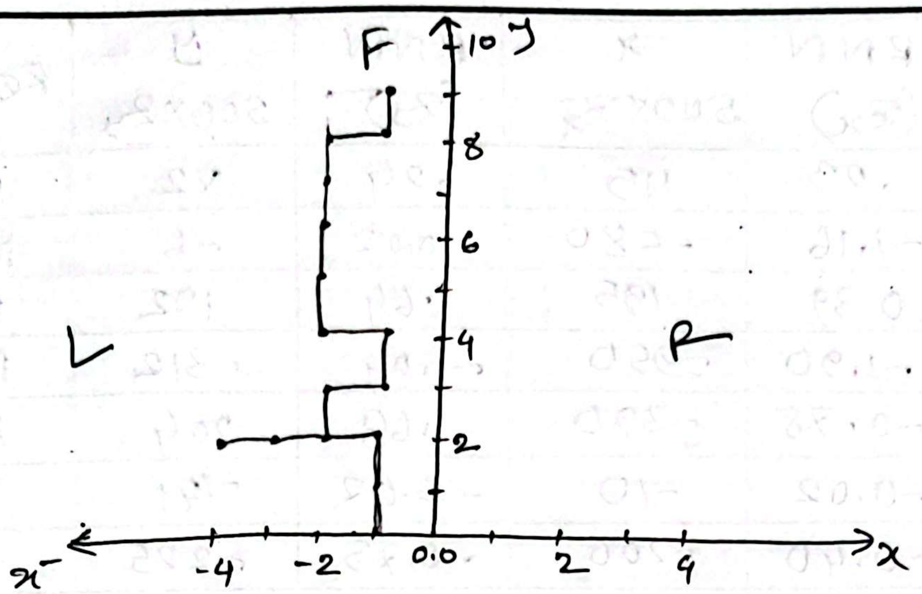
-1 in x axis and 4 in y axis

In 1st 20 steps, position (-1, 9)

Subject :

Date :

Time :



(2.7) Bombing Problem — RNN

A normal random variable X is given by

$$X = \mu + \sigma Z$$

$\mu \rightarrow$ true mean of distribution

$\sigma \rightarrow$ standard deviation of x

$z \rightarrow$ normal random number

$$\left.\begin{aligned} x &= \mu_x + \sigma_x Z_x \\ y &= \mu_y + \sigma_y Z_y \end{aligned}\right\} \text{coordinates}$$

∴ center point, $\mu_x = \mu_y = 0$

The given value of standard deviation in x and y directions are 500m and 300 m respectively.

$x_1 = 500 Z_x$ marked area

$y_1 = 300 Z_y$

Subject :

Date :

Time :

Bomb strike	RNN (Z_x)	x $500 \times Z_x$	RNN (Z_y)	y $300 \times Z_y$	Result
1	.23	115	.24	72	Hit
2	-1.16	-580	-0.02	-6	Miss
3	0.39	195	.64	192	Hit
4	-1.90	-950	-1.04	-312	Miss
5	-0.78	-390	.68	204	Hit
6	-0.02	-10	-0.47	-141	Hit
7	-0.40	-200	-0.75	-225	Hit
8	-0.66	-330	-0.44	-132	Hit
9	1.41	705	1.21	363	Miss
10	0.07	35	-0.08	-24	Hit
11	0.53	265	-1.56	-468	Miss
12	0.03	15	1.49	447	Miss
13	-1.19	-595	-1.19	-357	Miss
14	0.11	55	-1.86	-558	Miss
15	0.16	80	1.21	363	Miss
16	-0.82	-410	0.75	225	Hit
17	0.42	210	-1.50	-450	Miss
18	-0.89	-445	0.19	57	Hit
19	0.49	249	-1.44	-432	Miss
20	-0.21	-105	0.65	195	Hit

যখন x এর value 500 এবং y এর value 300 এর ভিতর হবে তখন Hit হবে। sign দেখার দরকার নেই শুধু নাম্বার compare করবো।
আর যেকোনো একটির value 500 বা 300 এর বাইরে হলে Miss হবে। আর দুইটিই বাইরে হলে Miss.

ship

Subject :

Date :

Time :

2.6 Reliability Problem

Bearing life (hours)	Probability	Cumulative Probability	Random digits assignment
1200	0.10	0.10	01–10
1400	0.25	0.35	11–35
1600	0.30	0.65	36–65
1800	0.25	0.90	66–90
2000	0.10	1.00	91–100
Extra ×	×	×	×

Delay time (minutes)	Probability	Cumulative Probability	Random digits assignment
2	0.3	0.3	1–3
5	0.5	0.8	4–8
8	0.2	1.0	9–0

one bearing (15 min), two (25 min), three (35 min)

Downtime costs USD.5 per min.

cost of technician USD.30 per ~~min~~ hour

" " per bearing USD.30

10000 hours

Bearing 1					Bearing 2					Bearing 3				
RD	Life	Clock	RD	Delay	RD	Life	Clock	RD	Delay	RD	Life	Clock	RD	Delay
25	1400	1400	9	8	65	1600	1600	5	5	94	2000	2000	7	5
82	1800	3200	8	5	91	2000	3600	0	8	87	1800	3800	0	8
16	1400	4600	0	8	30	1400	5000	5	5	63	1600	5400	6	5
14	1400	6000	5	5	66	1800	6800	6	5	66	1800	7200	2	2
82	1800	7800	2	2	32	1400	8200	5	5	30	1400	8600	2	2
45	1600	9400	3	2	29	1400	9600	3	2	69	1800	10400	7	5
09	1200	10600	7	5	08	1200	10800	2	2					
Total Delay 35					Total Delay 32					Total Delay 27				

20 bearings have been replaced.

cost of bearings $= 20 \times 30 = 600$

Cost of down time $= (35+32+27+(20\times 15)) \times 5 = 1970$

cost of technician $= (20 \times 15) \times \frac{30}{60} = 150$

total → 2720

Friendship

CamScanner

Subject :

Date :

Time :

Bearing 1 life (hours)	Bearing 2 life (hours)	Bearing 3 life (hours)	First failure (Hours)	Cumulated life (hours)	RD	Delay (min)
1400	1600	2000	1400	1400	4	5
1800	2000	1800	1800	3200	6	5
1400	1400	1600	1400	4600	7	5
1400	1800	1800	1400	6000	3	2
1800	1400	1400	1400	7400	4	5
1600	1400	1800	1400	8800	6	5
1200	1200	37/1600	1200	10000	7	5
				Total Delay		32

~~[illegible] bearings have been replaced.~~

cost of bearings $= 21 \times 30 = 630$

cost of down time $= (32 + (7 \times 3\cancel{4}5)) \times 5 = 1385$

cost of technician $= 7 \times 3\cancel{4}5 \times \frac{30}{60} = 122.5$

total 2137.5

(same operation time 7 replacements of bearings will occur, requiring a total of 21 bearings.)

The proposed repair policy is thus much more economical and will result in a savings 582.5 over a period of 10,000 hours of machine operation.

Subject : Chapter - 03

Date :
Time :

(3.2) A pure pursuit problem

Distance $Dist(t) = \sqrt{(yb(t) - yf(t))^2 + (xb(t) - xf(t))^2}$

↓ between fighter and bomber

$\sin\theta = \dfrac{yb(t) - yf(t)}{Dist(t)}$ $\cos\theta = \dfrac{xb(t) - xf(t)}{Dist(t)}$

(angle θ) between fighter and bomber

Speed/velocity, S = 20 km/min

Time limit = 12 min

Initial position = (0, 50) (xf, yf)

$xf(t+1) = xf(t) + S\cos\theta$

$yf(t+1) = yf(t) + S\sin\theta$

t	xb	yb	xf	yf	Dist(t)	$\cos\theta$	$\sin\theta$	xf(t+1)	yf(t+1)
0	100	0	0	50	111.803	0.894	-0.447	17.88	41.06
1	100	3	17.88	41.06	90.5	0.907	-0.420	35.94	32.66
2	120	6	35.94	32.66	88.18	0.953	-0.302	55.00	26.61
3	129	10	55.004	26.61	75.837	0.975	-0.219	74.518	22.231
4	140	15	74.518	22.231	65.88	0.993	-0.109	94.397	20.035
5	149	20	94.397	20.035	54.60	1.00	-0.0006	114.398	20.022
6	158	26	114.398	20.022	44.00	0.990	0.544	134.217	30.921
7	168	32	134.217	30.921	33.80	0.999	0.179	154.206	34.518
8	179	37	154.204	34.518	24.91	0.995	0.099	174.112	36.510
9	188	34	174.112	36.510	14.11	0.984	-0.177	193.79	32.952
10	198	30	193.79	32.952	5.141	0.818	-0.574	210.168	21.467
11	209	27	210.168	21.467	5.656	0.206	0.978	206.036	41.038
12	219	23	206.168	41.038	22.21	0.578	-0.821	217.728	24.618
13	226	19	217.728	24.618	9.999	0.827	-0.562	234.268	13.328

Subject :

Date :

Time :

Here, the distance between the persuer and the bomber is less than 10 Km after 10 minutes of persuing. So, the bomber will shoot.

3.3 Chemical Reaction

Two chemicals Ch1 and Ch2 produce Ch3

c_1, c_2, c_3 are the amounts of Ch1, Ch2, Ch3

Increase rates,

$$\frac{dc_1}{dt} = k_2 c_3 - k_1 c_1 c_2$$

$$\frac{dc_2}{dt} = k_2 c_3 - k_1 c_1 c_2 \qquad (k_1, k_2 \text{ constants})$$

$$\frac{dc_3}{dt} = 2k_1 c_1 c_2 - 2k_2 c_3$$

Now,

Quantities

$$c_1(t+\Delta t) = c_1(t) + \frac{dc_1(t)}{dt}\Delta t$$

$$c_2(t+\Delta t) = c_2(t) + \frac{dc_2(t)}{dt}\Delta t$$

$$c_3(t+\Delta t) = c_3(t) + \frac{dc_3(t)}{dt}\Delta t$$

Taking $c_1(0)$, $c_2(0)$ and $c_3(0)$,

$$c_1(\Delta t) = c_1(0) + [k_2 c_3(0) - k_1 c_1(0) \cdot c_2(0)]\Delta t$$

$$c_2(\Delta t) = c_2(0) + [k_2 c_3(0) - k_1 c_1(0) \cdot c_2(0)]\Delta t$$

riendship

Subject :

Date :
Time :

$$C_3(\Delta t) = C_3(0) + \left[2k_1 C_1(0) \cdot C_2(0) - 2k_2 C_3(0)\right] \Delta t$$

Now, using the state of the $2\Delta t$, $3\Delta t$ and so on, will be computed.

Chemical reaction,

$k_1 = 0.025$, $k_2 = 0.01$, time = 15.00

$C_1(0) = 50.0$, $C_2(0) = 25.0$, $C_3(0) = 0.0$

Time	C_1	C_2	C_3
0.00	50.00	25.00	0.00
0.1	46.88	21.88	6.25
0.2	44.32	19.32	11.36
0.3	42.19	17.19	15.62
0.4	40.39	15.39	19.22
0.5	38.86	13.86	22.29
0.6	37.53	12.53	24.93
0.7	36.38	11.38	27.24
0.8	35.37	10.37	29.25
0.9	34.49	9.49	31.03
~~0.10~~ 1.0	33.70	8.70	32.60
~~0.11~~ 1.1	33.80	8.00	34.00
~~0.12~~ 1.2	32.37	7.37	35.25

Sorry R korte পারবোনা, tired.

আগেরটার value use করে পরেরটা বের করতে হবে, time 0.00 এর C_1, C_2, C_3 দেওয়া আছে। এভাবে continue ---- until time = 15.00.

Subject :   Date :
                                                         Time :

(3.6) A serial chase problem

$$\begin{bmatrix} V_a = 35\,km/hr & V_c = 15\,km/hr \\ V_b = 25\,km/hr & V_d = 10\,km/hr \end{bmatrix}$$

[A(10,10), B(30,10), C(30,30), d(10,30)]

A → B
B → C
C → D
D → C

$\Delta t = 0.1$ ~~করে বাড়বে~~

$V_a\_step = 35 \times 0.1 = V_a \times dt$
$= 3.5\ km$

$V_b\_step = 25 \times 0.1 = 2.5\ km$

$V_c\_step = 15 \times 0.1 = 1.5\ km$

$V_d\_step = 10 \times 0.1 = 1.0\ km$

এখন থেকে প্রতিবার সময় পরিবর্তনের সাথে step ও change হবে।

$$dist_{AB} = \sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}$$

$$dist_{BC} = \sqrt{(C_x - B_x)^2 + (C_y - B_y)^2}$$

$$dist_{CD} = \sqrt{(D_x - C_x)^2 + (D_y - C_y)^2}$$

(যখন CD(distance) less than 0.005, $dist_{CD} < 0.005$ — hit করবে।)

যদি, তাহলে

যদি, target এর x-value > current → x বাড়বে
" " x – " < " → x কমাবে
" " y – " > " → y বাড়বে
" " y – " < " → y কমাবে

New Position of A, B, C, D.

New positions,

~~পূর্বের~~ আগের $\pm V \times \Delta t \times \frac{\Delta x}{dist} \rightarrow \cos\theta \rightarrow x_{new}$

আগের ~~পজিশন~~ $\pm V \times \Delta t \times \frac{\Delta y}{dist} \rightarrow \sin\theta \rightarrow y_{new}$
"

Subject :

Date :
Time :

A_x	A_y	B_x	B_y	C_x	C_y	D_x	D_y	Dist AB	Dist BC	Dist CD	t
10.0	10.0	30.0	10.0	30.0	30.0	10.0	30.0	20.0	20.0	20.0	0.0
13.5	10.0	30.0	12.5	28.5	30.0	11.0	30.0	16.68	17.56	17.5	0.1
16.96	10.52	29.79	14.99	27.0	30.0	12.0	30.0	13.58	15.26	15.0	0.2
20.27	11.68	29.33	17.45	25.5	30.0	13.0	30.0	10.74	13.12	12.50	0.3
23.22	13.56	28.60	19.84	24.0	30.0	14.0	30.0	8.27	11.15	10.0	0.4
25.49	16.21	27.57	22.12	22.5	30.0	15.0	30.0	6.25	9.37	7.50	0.5
26.66	19.52	26.22	24.22	21.0	30.0	16.0	30.0	4.72	7.78	5.00	0.6
26.33	23.0	24.54	26.08	19.5	30.0	17.0	30.0	3.55	6.38	2.50	0.7
24.57	26.03	22.57	27.61	18.0	30.0	18.0	30.0	2.55	5.15	0.00	0.8

$dist_{CD} < 0.005 \rightarrow t = 0.8$ hour $\rightarrow$ hit

3.7 Simulation of Exterior Ballistics

Subject : Chapter-4

Date :
Time :

4.1 Procedure of random number generator.

Sample drawn from continuous uniform distribution in interval 1 and 0.

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 \end{cases}$$

$$E(R) = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

$$V(R) = \int_0^1 x^2 dx - [E(R)]^2 = \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{1}{2}\right]^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

4.3 4.5 Pseudo random numbers (generator)

4.1) Mid square method:

→ Squaring the number

→ Extracting the middle digits

→ Using those middle digits as the next seed.

2061 → 2477 → 1355 → 8360 → 8896 → 1388 → 9265 → 8402 → 5936 → 2360 → 5696 → 4444

4.6 Congruence method or residue method

$$X_{n+1} = (aX_n + c) \bmod m$$ (Linear Congruential Generator)

X_{n+1} → next random number; a → Multiplier; X_n → current random number; c → Increment; mod → remainder; m → modulus

$$X_{n+1} = (aX_n) \bmod m$$ (Multiplicative Congruential Generator no inc.)

$$r_{i+1} = (ar_i + b) \bmod m$$ ← (खंदितर rule)

Subject :

Date :
Time :

Ex-4.1

$r_{i+1} = (ar_i + b) \bmod m$

a) mixed multiplicative congruential (MMC) Generator

taking, $a=13$, $b=1$ and $m=19$

let, $r_0 = 1$.

$r_1 = (1 \times 13 + 1) \bmod 19 = 14 \bmod 19 = 0$ residue $14 = 14$

$r_2 = (14 \times 13 + 1) \bmod 19 = 183 \bmod 19 = \cancel{(183 - 19 \times 9) \bmod 19}$

$= 9$ residue $12 = 12$

$\underbrace{(183 - 19 \times 9)}_{12}$

$r_3 = (12 \times 13 + 1) \bmod 19 = 157 \bmod 19 = 8$ residue $5 = 3$

$r_4 = (3 \times 13 + 1) \bmod 19 = 40 \bmod 19 = 2$ residue $2 = 1$

$\cancel{r_5 = 1 \times 13 + 1}$

This sequence of random numbers between 0 and 4 both inclusive will continue repeating.

$r_1 = (1 \times 13 + 1) \bmod 19 = 14 \bmod 19 =$ residue 14

$r_2 = (14 \times 13 + 1) \bmod 19 = 183 \bmod 19 =$ residue 12

$r_3 = (12 \times 13 + 1) \bmod 19 = 157 \bmod 19 =$ residue 5

$r_4 = (5 \times 13 + 1)$ " $19 = 66 \bmod 19 =$ residue 9

$r_5 = (9 \times 13 + 1)$ " $19 = 118$ " $19 =$ " 4

$r_6 = (4 \times 13 + 1)$ " $19 = 53$ " $19 =$ " 15

$r_7 = (15 \times 13 + 1)$ " $19 = 196$ " $19 =$ " 6

$r_8 = (6 \times 13 + 1)$ " $19 = 79$ " $19 =$ " 3

$r_{n+1} = (r_n \times a + b) \bmod m$

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Subject : Date : Time :

$r_9 = (3 \times 13 + 1) \bmod 19 = 40 \bmod 19 =$ residue 2

$r_{10} = (2 \times 13 + 1)$ " $19 = 27$ " $19 =$ " 8

$r_{11} = (8 \times 13 + 1)$ " $19 = 105$ " $19 =$ " 10

$r_{12} = (10 \times 13 + 1)$ " $19 = 131$ " $19 =$ " 17

$r_{13} = (17 \times 13 + 1)$ " $19 = 222$ " $19 =$ " 13

$r_{14} = (13 \times 13 + 1)$ " $19 = 170$ " $19 =$ " 18

$r_{15} = (18 \times 13 + 1)$ " $19 = 235$ " $19 =$ " 7

$r_{16} = (7 \times 13 + 1)$ " $19 = 92$ " $19 =$ " 16

$r_{17} = (16 \times 13 + 1)$ " $19 = 209$ " $19 =$ " 0

$r_{18} = (0 \times 13 + 1)$ " $19 = 01$ " $19 =$ " 01

This sequence of random numbers between 0 and 18 both inclusive will continue repeating. The number 18, which is one less than modulus is called Euler function.

R.N. between 0 and 1,

$$R_i = \frac{r_i}{m}, \quad i = 1, 2, 3, \ldots$$

$R_1 = \frac{14}{19} = 0.737$ $\quad R_2 = \frac{12}{19} = 0.632$ $\quad R_3 = \frac{5}{19} = 0.263$

$R_4 = \frac{9}{19} = 0.474$ $\quad R_5 = \frac{4}{19} = 0.211$ $\quad R_6 = \frac{15}{19} = 0.789$

etc..

Subject :

Date :
Time :

b) Multiplicative Congruential (MC) Generator:

* $r_{i+1} = a r_i \bmod m$

taking, $a=13$, $m=19$ and seed $r_0=1$

$r_1 = 1\times13 \bmod 19 = 13 \bmod 19$ = residue 13
$r_2 = 13\times13 \bmod 19 = 169$ " $19 =$ " 17
$r_3 = 17\times13 \bmod 19 = 221$ " $19 =$ " 12
$r_4 = 12\times13$ " $19 = 156$ " $19 =$ " 4
$r_5 = 4\times13$ " $19 = 52$ " $19 =$ " 14
$r_6 = 14\times13$ " $19 = 182$ " $19 =$ " 11
$r_7 = 11\times13$ " $19 = 143$ " $19 =$ " 10
$r_8 = 10\times13$ " $19 = 130$ " $19 =$ " 16
$r_9 = 16\times13$ " $19 = 208$ " $19 =$ " 18
$r_{10} = 18\times13$ " $19 = 234$ " $19 =$ " 6
$r_{11} = 6\times13$ " $19 = 78$ " $19 =$ " 2
$r_{12} = 2\times13$ " $19 = 26$ " $19 =$ " 7
$r_{13} = 7\times13$ " $19 = 91$ " $19 =$ " 15
$r_{14} = 15\times13$ " $19 = 195$ " $19 =$ " 5
$r_{15} = 5\times13$ " $19 = 65$ " $19 =$ " 8
$r_{16} = 8\times13$ " $19 = 104$ " $19 =$ " 9
$r_{17} = 9\times13$ " $19 = 117$ " $19 =$ " 3
$r_{18} = 3\times13$ " $19 = 39$ " $19 =$ " 1

The sequence of numbers obtained is 1, 13, 17, 12, 4, 14, 11, 10, 16, 18, 6, 2, 7, 15, 5, 8, 9, 3, 1, which will repeat forever.

Subject :

Date :
Time :

c) Additive Congruential Generator,

$\star\ R_{i+1} = (R_i + b) \mod m$

taking, $m = 19$, $b = 11$, let, $R_0 = 1$

$R_1 = (1+11) \mod 19 = 12 \mod 19 = 12$

$R_2 = (12+11)$ " $19 = 23$ " $19 = 4$

$R_3 = (4+11)$ " $19 = 15$ " $19 = 15$

$R_4 = (15+11)$ " $19 = 26$ " $19 = 7$

$R_5 = (7+11)$ " $19 = 18$ " $19 = 18$

$R_6 = (18+11)$ " $19 = 29$ " $19 = 10$

$R_7 = (10+11)$ " $19 = 21$ " $19 = 2$

$R_8 = (2+11)$ " $19 = 13$ " $19 = 13$

$R_9 = (13+11)$ " $19 = 24$ " $19 = 5$

$R_{10} = (5+11)$ " $19 = 16$ " $19 = 16$

$R_{11} = (16+11)$ " $19 = 27$ " $19 = 8$

$R_{12} = (8+11)$ " $19 = 19$ " $19 = 1$

4.7 Arithmetic Congruential Generator (pseudo)

$\star\ R_{i+1} = (R_{i-1} + R_i) \mod m$

If, $R_1 = 9$, $R_2 = 13$ and $m = 17$

$R_3 = (9+13) \mod 17 = 22 \mod 17 = 5$

$R_4 = (13+5) \mod 17 = 18$ " $17 = 1$

$R_5 = (5+1)$ " $17 = 6$ " $17 = 6$

$R_6 = (1+6)$ " $17 = 7$ " $17 = 7$

$R_7 = (6+7)$ " $17 = 13$ " $17 = 13$

Friendship

Subject :

Date :

Time :

$r_8 = (7+13) \bmod 17 = 20 \bmod 17 = 3$

and so on ~~as 1, 3, 4, 7, 11, 13,~~ - - - - -

This results into quite a long sequence.

(4.8) Combined Congruential Generators

$$r_i = \left(\sum_{j=1}^{k} (-1)^{j-1} r_{i,j} \right) \text{mode } m_1 - 1$$

$$R_i = \begin{cases} \dfrac{r_i}{m_i}, & r_i > 0 \\ \dfrac{m_i - 1}{m_i}, & r_i = 0 \end{cases}$$

Possible period,

$$P = \frac{(m_1-1)(m_2-1) \cdots (m_k-1)}{2^k - 1}$$

(4.10) Uniformity test

(1) Kolmogorov-Smirnov Test

(2) Chi-squared test

Ex-4.2

$\alpha = 0.01$, $N = 10$, critical value $= 0.368$

$\alpha = 0.05$, $N = 10$, critical value $= 0.410$

Given R.Ns,

.24, .89, .11, .61, .23, .86, .41, .64, .50, .65

Subject :

Date :
Time :

	1	2	3	4	5	6	7	8	9	10
R_i	.11	.23	.24	.41	.50	.61	.64	.65	.86	.89
i/N; i=1	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.0
D^+ $(i/N)-R_i$	—	—	.06	—	0	—	.06	.15	.04	.11
D^- $R_i-((i-1)/N)$	.11	.13	.04	.11	.10	.11	.04	—	.06	—

$$D^+ = \max_{1\le i\le N}\left\{\frac{i}{N}-R_i\right\} \qquad D=\max(D^+,D^-)$$

$$D^- = \max_{1\le i\le N}\left\{R_i-\left(\frac{i-1}{N}\right)\right\}$$

$D^+ = 0.15$, $D^- = 0.13$ $\therefore D = 0.15$

$D=0.15 <$ both critical values, we fail to reject the null hypothesis.

(4.12) Chi-Squared Test

Divide the 100 numbers into 10 equal intervals.

$E=\frac{100}{10}=10$

Class	Count	Frequency O	Diff (O−E)	Diff² $(O-E)^2$	$\frac{(O_i-E)^2}{E}$
$0<r\le10$	IIII IIII IIII	13	3	9	0.9
$10<r\le20$	IIII IIII	8	2	4	0.4
$20<r\le30$	IIII IIII	9	1	1	0.1
$30<r\le40$	IIII IIII IIII	13	3	9	0.9
$40<r\le50$	IIII II	7	3	9	0.9
$50<r\le60$	IIII IIII IIII	13	3	9	0.9
$60<r\le70$	IIII	5	5	25	2.5
$70<r\le80$	IIII IIII II	12	2	4	0.4
$80<r\le90$	IIII IIII II	12	2	4	0.4
$90<r\le100$	IIII III	8	2	4	0.4
		100		total = 78	7.8

Subject :

Date :

Time :

$\therefore$ Chi-square $= \frac{78}{10} = 7.8$

For $(10-1) = 9$ degress of freedom, at 95% confidence level, the acceptable value of chi-square is 16.9, which is more than 7.8. The random numbers are acceptable at 95% confidence level. Uniformity of distribution is not rejected.

[illegible]

.4.13, .49, .95, .82, .19, .41, .31, .12, .53, .62, .40, .87, .83, .26, .01, .91, .55, .38, .75, .90

These 18 random numbers giving 18 pairs, are grouped into 3 classes with expectation of 6 in each group.

Class	Count	Frequency	Diff	$(Diff)^2$
$R_1 \le .33$ & $R_2 \le 0.33$	**	2	4	16
$R_1 \le .62$ & $R_2 \le 0.62$	****~~****~~**	6	0	0
$R_1 \le 1.0$ & $R_2 \le 1.0$	**********	10	4	16
		18		32

Chi square $= \frac{32}{6} = 5.33$

$(6-1)$ 5 degress of freedom, $\alpha = 0.05$,
11.070

$x^2 <$ degress of freedom $\rightarrow$ uniformity distributed

$x^2 <$ " " " $\rightarrow$ not serially auto correlated

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Subject : Date : Time :

(4.14) Poker Test

* 4-digit random number → (10000 generated)

Expected calculation (Probability) (Expected)

	Probability	Expected
(1) All digits different	0.5040	5040
(2) One pair	0.4320	4320
(3) Two "	0.0540	540
(4) Three of a kind	0.0090	90
(5) Four of a kind	0.0010	10

* 5-digit random number →

	Probability	Expected
(1) All 5 digits different	0.30240	3024
(2) one pair	0.50400	5024
(3) Two pairs	0.10800	1080
(4) Three of a kind	0.07200	720
(5) Full houses	0.00900	90
(6) four of a kind	0.00450	45
(7) five of a kind	0.00010	1

* 3-digit random number →

	Probability	Expected
(1) All 3 digits different	0.72	7200
(2) One pair	0.27	2700
(3) Three of a kind	0.01	100

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Subject :

Date :

Time :

Ex-4.5

Combination distribution i	Observed Distribution O_i	Expected E_i	$\frac{(O_i-E_i)^2}{E_i}$
Five different digits	3025	3024	0.86
Pairs	4935	5040	2.19
Two-pairs	1135	1080	1.88
Three of a kind	695	720	0.82
Full houses	105	90	2.5
Four of a kind	54	45	1.8
Five of a kind	01	01	0.0
	10,000	10,000	10.1
x	^	^	^

The appriate degress of freedom in this case are 6, one less than the number o Combinations.

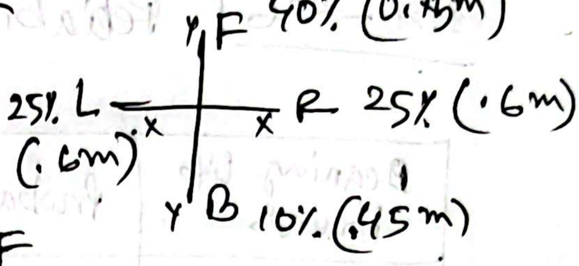
critical value, 16.8 $\rightarrow \alpha = 0.01$

$16.8 > x^2$

critical value $> x^2 \rightarrow$ hypothesis of independance cannot be rejected on the basis of Poker test.

critical value $< x^2 \rightarrow$ hypothesis of independance of the numbers is rejected.

Ex-2.5 Random walk



Direction	Probability	R.N	
F	0.4	0,1,2,3	F
B	0.1	4	B
R	0.25	5,6,7	R
L	0.25	8,9	L

step	R.N	Direction	X	Y
1	6	R	0.6	0
2	2	F	0.6	0.~~4~~75
3	0	F	0.6	~~0.8~~ 1.5
4	6	R	1.2	~~0.8~~ 1.5
5	8	L	0.6	~~0.8~~ 1.5
6	4	B	0.6	1.05
7	5	R	1.2	1.05
8	9	L	0.6	1.05
9	3	F	0.6	1.8
10	2	F	0.6	2.55
11	2	F	0.6	3.3
12	1	F	0.6	4.05
13	7	R	1.2	4.05
14	8	L	0.6	4.05
15	7	R	1.2	4.05
16	6	R	1.8	4.05
17	1	F	1.8	4.8
18	9	L	1.2	4.8
19	0	F	1.2	5.55
20	2	F	1.2	6.3

1.2 in x axis &
6.3 in y axis } distance