

a) Define the following terminologies: i) System ii) Model iii) Simulation iv) Analytical solution. Demonstrate different ways in which a system might be studied.

- **System:** A system is defined to be a collection of entities, e.g., people or machines, that act and interact together toward the accomplishment of some logical end.

Two types of system:

- ✓ **Discrete system:** The system in which the state changes abruptly at discrete points in time are called discrete systems.

Example: A bank is an example of a discrete system.

- ✓ **Continuous system:** System in which the state of the system changes continuously with time are called continuous system.

Example: An airplane moving through the air is an example of a continuous system.

- **Model:** A model is a simplified representation of a real-world system or process used to understand or predict how it behaves.

Example: A toy car can be a model of a real car to show how it moves.

- ✓ **Physical Model:** A physical model is a tangible, small-scale version of something larger. Like a model airplane that looks and behaves like a real airplane.

- ✓ **Mathematical Model:** A mathematical model uses equations and formulas to represent how a system works. A simple equation like $d=rt$ can be used to represent the relationship between distance, rate (or speed), and time.

- **Simulation:** Simulation is the imitation of reality. Example: Flight simulator.

- **Analytical solution:** An analytical solution is a precise, exact answer to a problem derived using mathematical methods.

Example: Solving the equation $x+2=5$ to get $x=3$.

★ Different ways in which a system might be studied.

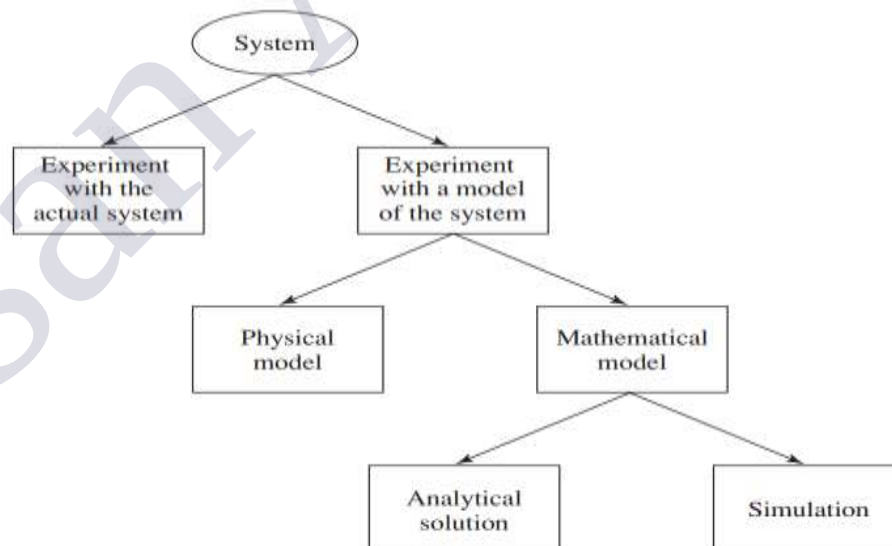


FIGURE 1.1
Ways to study a system.

✧ Analytical Solution vs. Simulation

TABLE 1.1
Comparison between simulation and analytical methods

Attributes	Simulation Based Methods	Analytical Methods
Computation Time	Time consuming (depends on number of iterations)	Usually faster than simulation
Final Results	Approximate	Exact / Precise
Complexity Consideration	It would be easier to evaluate complex systems through simulation	Simplification is needed for large-scale systems
Further Results Extraction	Only statistical calculation	It is possible to drive other factors like MTTF/MTBF from results directly

✧ Deterministic vs. Stochastic Simulation Models.

Stochastic Models	Deterministic Models
Accounts for uncertainty and randomness	No uncertainty or randomness
Random variation in the inputs	No random inputs
Estimate of probability of various outcomes	One solution to a specific set of values
Different results every time	Results are always the same

✧ Continuous vs. Discrete Simulation Models.

Items	Continuous models in time	Discrete models in time
System to be simulated	Mechanical units of complex function	Multiple entities, usually simple
State of the System	Energy distribution within the system	System properties characterization
State of the System Measures	Position, speed (deterministic)	Length of line, frequency (statistical properties)
Updating factor	Time	Transition between states
Simulation capacity	Analyses mechanical units deterministic behaviour	Analyses the stochastic nature of the interactions between entities
Regular Applications	Unit project and analysis; real time training	Analyses and plans operations with multiple entities

- b) What are the steps of simulation for a single-server queuing system? Describe the problem statement and intuitive explanation of a single-server queuing system.

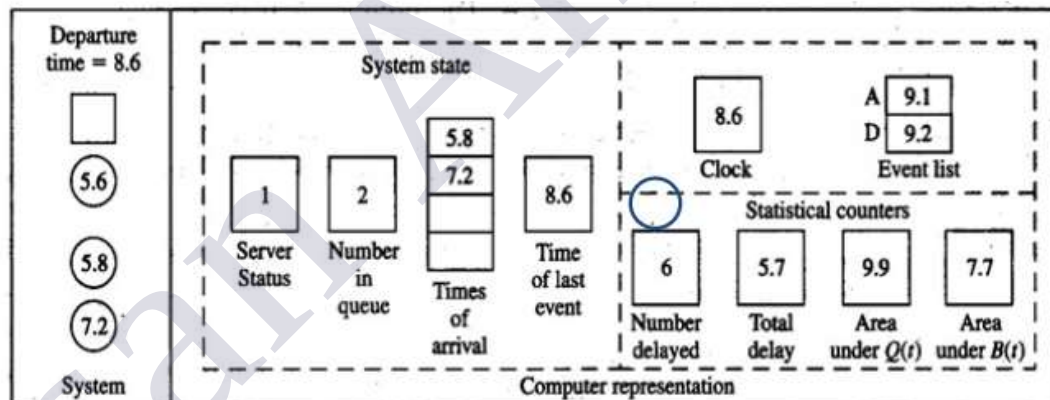
[Click here and check the slide for complete answer.](#)

1. Problem statement:

- Recall single-server queuing model
- Assume interarrival times are independent and identically distributed (IID) random variables
- Assume service times are IID, and are independent of interarrival times
- Queue discipline is FIFO
- Start empty and idle at time 0
- First customer arrives after an interarrival time, not at time 0
- Stopping rule: When n th customer has completed delay in queue (i.e., *enters service*) ... n will be specified as input



2. Intuitive explanation.



Interarrival times: ~~0.4~~, ~~1.2~~, ~~0.6~~, ~~1.7~~, ~~0.2~~, ~~1.6~~, ~~0.2~~, ~~1.4~~, ~~1.9~~, ...

Service times: ~~2.0~~, ~~0.7~~, ~~0.2~~, ~~1.1~~, ~~3.7~~, ~~0.6~~, ...

Final output performance measures:

Average delay in queue = $5.7/6 = 0.95$ min./cust.

Time-average number in queue = $9.9/8.6 = 1.15$ custs.

Server utilization = $7.7/8.6 = 0.90$ (dimensionless)

3. Program organization and logic. (First two step for second part question answer)

- How to “draw” (or generate) an observation (*variate*) from an exponential distribution?
- Proposal:
 - Assume a perfect *random-number generator* that generates IID variates from a continuous uniform distribution on $[0, 1]$...
 - Algorithm:
 1. Generate a random number U
 2. Return $X = -\beta \ln U$
 - Proof that algorithm is correct

$$\begin{aligned}
 P(\text{generated } X \leq x) &= P(-\beta \ln U \leq x) \\
 &= P(\ln U \geq -x/\beta) \\
 &= P(U \geq e^{-x/\beta}) \\
 &= P(e^{-x/\beta} \leq U \leq 1) \\
 &= 1 - e^{-x/\beta}
 \end{aligned}$$

4. **C Program:** C program for simulating an M/M/1 queue, emphasizing the use of clear variable names and function prototyping. It also notes that floating-point inaccuracies in different compilers can affect simulation results, especially when events are close in time.
5. **Simulation Output and Discussion:** The results of an M/M/1 queue simulation, highlighting that the output numbers, such as average delay, number in the queue, and server utilization, are influenced by random number generation and input parameters. It discusses the challenges of choosing a stopping rule for accurate steady-state estimation and suggests focusing on the delay in the queue rather than the total waiting time for efficiency in estimating system performance.
6. **Alternative stopping rules:** The text describes modifying a queueing simulation to stop after a fixed amount of time, such as 8 hours, by introducing a dummy "end-simulation" event. This event, scheduled to occur at the specified time, ensures that the simulation stops when the event's time is reached. The changes involve adjusting the program's external definitions, main function, and initialization and reporting functions to handle this fixed run length and manage the end of the simulation properly.
7. **Determining the Events and Variables:** The text discusses the event-graph method for modeling complex systems, where events are represented as nodes connected by arrows that show how they affect each other. It explains that this technique helps simplify the model by eliminating unnecessary events and ensuring correct initialization. The method is useful for managing and visualizing event sequences in simulations, especially in complex systems, and can also help detect errors and predict computational demands.

❖ Steps in sound simulation study.

